CHAPTER 26

THE ARBITRAGE-FREE VALUATION FRAMEWORK

1. (C) $101.85

Explanation

Path 3 value =

$$\frac{3.0}{(1.02)} + \frac{3.0}{(1.02)(1.02078)} + \frac{103.0}{(1.02)(1.02078)(1.030216)} = 101.85$$

(Module 26.2 LOS 26.g)

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2. (B) $102.20

Explanation

Value =

$$\frac{1.50}{\left[1+\frac{0.0125}{2}\right]^{1}} + \frac{1.50}{\left[1+\frac{0.013}{2}\right]^{2}} + \frac{1.50}{\left[1+\frac{0.018}{2}\right]^{3}} + \frac{1.50}{\left[1+\frac{0.02}{2}\right]^{4}} + \frac{1.50}{\left[1+\frac{0.022}{2}\right]^{5}} + \frac{101.50}{\left[1+\frac{0.0225}{2}\right]^{6}}$$

(Module 26.1 LOS 26.b)

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3. (C) the cash flows for the MBS are dependent upon the path that interest rates follow.

Explanation

A binomial model or any other model that uses the backward induction method cannot be used to value an MBS because the cash flows for the MBS are dependent upon the path that interest rates have followed.

(Module 26.2, LOS 26.h)

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4. (C) **$100.18**

Explanation

Path 1 value =

$$\frac{3.0}{(1.02)}+\frac{3.0}{(1.02)(1.02805)}+\frac{103.0}{(1.02)(1.02805)(1.040787)}=100.18$$

(Module 26.2, LOS 26.g)

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5. (C) **4.63%**

Explanation

Lower node interest rate = 6.25 / $e^{2\times0.15}$ = 4.63%

(Module 26.1, LOS 26.c)

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6. (C) **$102.58**

Explanation

Path 4 value =

$$\frac{3.0}{(1.02)}+\frac{3.0}{(1.02)(1.02078)}+\frac{103.0}{(1.02)(1.02078)(1.022384)}=102.58$$

(Module 26.2, LOS 26.g)

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7. (B) **Ho-Lee model.**

Explanation

Ho-Lee model is an arbitrage-free term structure that is calibrated to the current actual term structure (regardless of whether it is upward or downward sloping). Vasicek and Cox-Ingersoll-Ross model are examples of equilibrium term structure models and may generate term structures inconsistent with current market observations.

(Module 26.3, LOS 26.i)

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8. (B) **$101.15**

Explanation

Path 2 value =

$$\frac{3.0}{(1.02)} + \frac{3.0}{(1.02)(1.02805)} + \frac{103.0}{(1.02)(1.02805)(1.030216)} = 101.15$$

(Module 26.2, LOS 26.g)

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9. (B) **99.81.**

Explanation

The option-free bond price tree is as follows:

100.00

A ==> 99.74

99.81 100.00

100.16

100.00

As an example, the price at node A is obtained as follows:

Price_A = (prob x (P_{up} + coupon/2) + prob × (P_{down} + coupon/2))/(1 + rate/2)

= (0.5 x (100 + 3) + 0.5 x (100 + 3))/(1 + 0.0653/2) = 99.74.

The bond values at the other nodes are obtained in the same way.

The calculation for node 0 or time 0 is

0.5[(99.74 + 3)/(1+ 0.063/2) + (100.16 + 3)/(1 + 0.063/2)] = 99.81

(Module 26.1, LOS 26.e)

Related Material

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10. (A) **102.659.**

Explanation

The tree will have three nodal periods: 0, 1, and 2. The goal is to find the value at node 0. We know the value in nodal period 2: V_2 =100. In nodal period 1, there will be two possible prices:

$V_{1,U}$ = [(100+10)/1.09+(100+10)/1.09]/2= 100.917

$V_{1,L}$ = [(100+10)/1.08+(100+10)/1.08]/2= 101.852

Thus

V_0 = [(100.917+10)/1.085+(101.852+10)/1.085]/2= 102.659

(Module 26.1, LOS 26.e)

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11. (B) $93.15

Explanation

First we compute the spot rates:

S_1: (given) = 5%

S_2: 100 =

$$\frac{6.0}{(1.05)} + \frac{106.0}{(1+S_2)^2} \rightarrow S_2 = 6.03\%$$

S_3 :100 =

$$\frac{6.5}{(1.05)} + \frac{6.5}{(1.0603)^2} + \frac{106.5}{(1+S_3)^2} \rightarrow S_3 = 6.56\%$$

S_4: 100 =

$$\frac{7.0}{(1.05)} + \frac{7.0}{(1.0603)^2} + \frac{7.0}{(1.0656)^3} + \frac{107.0}{(1+S_4)^4} \rightarrow S_4 = 7.10\%$$

Then we use the spot rates to value the 4-year, 5% annual pay bond:

$$\text{Value} = \frac{5.0}{(1.05)^1} + \frac{5.0}{(1.0603)^2} + \frac{5.0}{(1.0656)^3} + \frac{105.0}{(1.071)^4} = 93.15$$

(Module 26.1, LOS 26.b)

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12. (A) Adjacent forward rates in a nodal period are one standard deviation apart.

Explanation

A binomial interest rate tree has two desirable properties: non-negative interest rates and higher volatility at higher rates. Additionally, adjacent forward rates in a nodal period are two standard deviations apart.

(Module 26.1, LOS 26.c)

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13. (A) more suitable when valuing securities whose cash flows are interest rate path dependent.

Explanation

Monte Carlo method does not require that cash flows of a security be path independent and hence is suitable alternative to the binomial model to value securities such as mortgage backed securities whose cash flows are path dependent. The model generating interest rates paths in a Monte Carlo simulation is based on an assumed level of volatility (i.e., model needs a volatility input). The model generating interest rates in a Monte Carlo simulation can incorporate bounds for interest rates to force mean reversion of rates. Such bounded optimization is not possible in a binomial model.

(Module 26.2, LOS 26.h)

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14. (C) **the same value.**

Explanation

Because these two valuation methods are arbitrage-free, the two values obtained must be the same. An option-free bond that is valued by discounting by the spot rates should have the same value as if the binomial interest rate tree was used.

(Module 26.2, LOS 26.f)

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15. (B) **98.67.**

Explanation

The option-free bond price tree is as follows:

		100.00
	A → 98.89	
98.67		100.00
	99.56	
		100.00

As an example, the price at nodes A is obtained as follows:

$Price_A$ = (prob x (P_{up} + coupon/2) + prob × (P_{down} + coupon/2))/(1 + rate/2)

= (0.5 x (100 + 2.5) + 0.5 x (100 + 2.5))/(1 + 0.0730/2) = 98.89.

The bond values at the other nodes are obtained in the same way.

The calculation for node 0 or time 0 is

0.5[(98.89 + 2.5)/(1+ 0.062/2) + (99.56 + 2.5)/(1 + 0.062/2)]

0.5(98.3414 + 98.9913) = 98.6663

(Module 26.1, LOS 26.e)

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16. (C) **Security valuations are not consistent with the value additivity principle.**

Explanation

If the principle of value additivity holds, it will not be possible to earn arbitrage profits through stripping (or reconstitution). If a portfolio of strips is trading for less than the price of an intact bond, one can purchase the strips, combine them ("reconstitution"), and sell them as a bond. Similarly, if the bond is worth less than its component parts, one could purchase the bond, break it into a portfolio of strips ("stripping"), and sell those components. When one security trades at a lower price than another security with identical characteristics, this is known as dominance, and the arbitrage required to earn a profit involves going long the underpriced security and short the overpriced security.

(Module 26.1, LOS 26.a)

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17. (A) **the corresponding interest rates and interest rate probabilities are used to discount the value of the bond.**

Explanation

For a bond that has N computing periods, the current value of the bond is determined by computing the bond's possible values at period N and working "backwards" to the present. The value at any given node is the probability-weighted average of the discounted values of the next period's nodal values.

(Module 26.1, LOS 26.e)

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18. (A) **6.3123%**

Explanation

Value represented by 'A' = 7.7099 / $e^{2x0.10}$ = 6.3123%

(Module 26.2, LOS 26.d)

Related Material

SchweserNotes - Book 4

19. (B) **Mean reversion of interest rates.**

Explanation

A binomial interest rate tree has two desirable properties: non-negative interest rates and higher volatility at higher rates. Binomial trees do not force mean reversion of rates.

(Module 26.1, LOS 26.c)

Related Material

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20. (B) **positively related to the current level of the short-term interest rate.**

Explanation

Under the Cox-Ingersoll-Ross model, the random or stochastic component incorporates the square root of current level of interest rate. Hence the higher the current level of interest rates, the higher the volatility of interest rates.

(Module 26.3, LOS 26.i)

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21. (C) **8.437%**

Explanation

Upper node interest rate = 6.25 x $e^{2x0.15}$ = 8.437%

(Module 26.1, LOS 26.c)

Related Material

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22. (B) **the current value of a bond based on possible final values of the bond.**

Explanation

Backward induction refers to the process of valuing a bond using a binomial interest rate tree. For a bond that has N compounding periods, the current value of the bond is determined by computing the bond's possible values at period N and working "backwards."

(Module 26.1, LOS 26.e)

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23. (A) **7.5835%**

Explanation

Value represented by 'B' = 7.7099 / $e^{2x0.10}$ = 7.5835%

(Module 26.2, LOS 26.d)

Related Material

SchweserNotes - Book 4

24. (A) **Cox-Ingersoll-Ross model.**

Explanation

The model given is an example of the Cox-Ingersoll-Ross model which differs from the Vasicek model by including the square root of current level of short-term interest rates in the stochastic part of the equation.

(Module 26.3, LOS 26.i)

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25. (A) **11.3132%**

Explanation

Value represented by 'C' = 9.2625 / $e^{2x0.10}$ = 11.3132%

(Module 26.2, LOS 26.d)

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26. (C) **statistical accuracy of the estimated value.**

Explanation

Increasing the number of paths would increase the statistical accuracy of the estimate but does nothing for the fundamental accuracy of the estimated value which depends on the quality of model inputs. Model utility depends on valuation accuracy of the model and hence would not increase as we increase the number of paths.

(Module 26.2, LOS 26.h)

Related Material

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27. (A) **The tree is not calibrated properly because it is not consistent with market prices.**

Explanation

The tree is not calibrated properly – it does not value 3-year 7% bond at par (i.e. the market price):

$$V_{2,UU} = \frac{107}{(1.13818)} = \$94.01$$

$$V_{2,UL} = \frac{107}{(1.092625)} = \$97.93$$

$$V_{2,LL} = \frac{107}{1.062088} = \$100.74$$

$$V_{1,U} = \frac{1}{1.08948} \times \left[\frac{94.01 + 97.93}{2} + 7\right] = \$94.51$$

$$V_{1,L} = \frac{1}{1.05998} \times \left[\frac{97.93 + 100.74}{2} + 7\right] = \$100.31$$

$$V_{0,} = \frac{1}{1.05} \times \left[\frac{94.51 + 100.32}{2} + 7\right] = \$99.44$$

The adjacent nodes in the binominal tree any nodal period are all two standard deviations apart.

(Module 26.2, LOS 26.d)

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28. (C) **The price of the bond is known at maturity.**

Explanation

The objective is to value a bond's current price while the bond price at maturity is known. Therefore, price at maturity is used as a starting point, and we work backward to the current value.

(Module 26.1, LOS 26.e)

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29. (C) **3%**

Explanation

The long-term expected value of short-term rates is the mean reverting level (b) estimated by Sebelius to be 3%.

(Module 26.3, LOS 26.i)

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30. (B) 32

Explanation

For a 3-year, semiannual coupon bond, there will be six nodal periods resulting in $2^{(6-1)}$ = 32 paths.

(Module 26.2, LOS 26.g)

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