CHAPTER 31

VALUATION OF CONTINGENT CLAIMS

1 (B) **risk-free rate.**

Explanation

The value of a put option is negatively related to increases in the risk-free rate.

(Module 31.7, LOS 31.k)

Related Material

SchweserNotes - Book 4

2. (A) **The yield curve for risk-free assets is fixed over the term of the option.**

Explanation

The yield curve is assumed to be flat so that the risk-free rate of interest is known and constant over the term of the option. Having a fixed yield curve does not necessarily imply that the yield curve is flat. BSM assumptions include that markets are frictionless (no taxes, transactions costs) and that the options are European-style, meaning that early exercise is not allowed.

(Module 31.6, LOS 31.f)

Related Material

SchweserNotes - Book 4

3. (A) **the right to enter a swap in the future as the fixed-rate payer.**

Explanation

A payer swaption give its holder the right to enter a swap in the future as the fixed-rate payer.

(Module 31.6, LOS 31.j)

Related Material

SchweserNotes - Book 4

4. (B) **Changes in volatility are known and predictable.**

Explanation

The BSM model assumes that volatility of the return is known and constant (i.e., not changing). Other assumptions of the BSM model include the continuously compounded risk-free interest rate is known and constant and the options are European (meaning that early exercise is not allowed).

(Module 31.6, LOS 31.f)

Related Material

SchweserNotes - Book 4

5. (A) $2,230

Explanation

Given the exercise rate of 2.00%, the put option has a positive payoff for nodes P^{--} and P^{-+}. The payoff at node P^{--} can be calculated as:

[Max (0, 0.02 - 0.0128)] x $ 1,000,000 = $7,200.

The payoff at node P^{-+} can be calculated as:

[Max (0, 0.02 – 0.019)] x $ 1,000,000 = $ 1,000.

Value at node P^{-} = [(0.5 x 7,200) + (0.5 x 1,000)] / (1.0155) = $4,037

Value at node P^{+} = [(0.5 x 0) + (0.5 x 1,000)] / (1.0231) = $489

And the value at node P = [(0.5 x 4,037) + (0.5 x 489)] / (1.015) = $2,230.

(Module 31.5, LOS 31.e)

Related Material

SchweserNotes - Book 4

6. (A) the risk free rate must be constant and known.

Explanation

The BSM model is not useful for pricing options on bond prices and interest rates. In those cases, interest rate volatility is a key factor in determining the value of the option. BSM can be modified to deal with cash flows like coupon payments. The assumption that "the price of the underlying asset follows a lognormal distribution" is not applicable.

(Module 31.6, LOS 31.f)

Related Material

SchweserNotes - Book 4

7. (C) 0.01.

Explanation

The gamma of an option is computed as follows:

Gamma = change in delta/change in the price of the underlying

= (0.7 — 0.6)/(110 — 100) = 0.01

(Module 31.7, LOS 31.k)

Related Material

SchweserNotes - Book 4

8. (C) must have the market price of the option.

Explanation

In order to compute the implied volatility we need the time to expiration, the exercise price, and the marketrisk-free rate, the current asset price, the price of the option.

(Module 31.7, LOS 31.n)

Related Material

SchweserNotes - Book 4

9. (A) **reference rate exceeds the strike rate.**

Explanation

An interest rate cap is a package of European-type call options (called caplets) on a reference interest rate.

(Module 31.6, LOS 31.j)

Related Material

SchweserNotes - Book 4

10. (B) **subtracting the present value of the dividend from the current stock price.**

Explanation

The option pricing formulas can be adjusted for dividends by subtracting the present value of the expected dividend(s) from the current asset price.

(Module 31.5, LOS 31.d)

Related Material

SchweserNotes - Book 4

11. (B) **$2,022**

Explanation

Given the exercise rate of 2.00%, the call option has a positive payoff for node C^{++} only. The payoff at node C^{++} can be calculated as:

[Max (0, 0.0284 — 0.02)] x $ 1,000,000 = $ 8,400.

Value at node C^{+} = [(0.5 x 8,400) + (0.5 x 0)] / (1.0231) = $4,105

Value at node C^{-} = 0

And the value at node C = [(0.5 x 4,105) + (0.5 x 0)] / (1.015) = $2,022

(Module 31.5, LOS 31.e)

Related Material

SchweserNotes - Book 4

12. (B) **dollar change in the option price divided by the dollar change in the stock price.**

Explanation

The delta of an option is the dollar change in option price per $1 change in the price of the underlying asset.

(Module 31.7, LOS 31.k)

Related Material

SchweserNotes - Book 4

13. (B) **implied volatility.**

Explanation

The question describes the process for finding the expected volatility implied by the market price of the option.

(Module 31.7, LOS 31.n)

Related Material

SchweserNotes - Book 4

14. (B) **a put option on the stock and sell a call option on the stock.**

Explanation

Buying a put option and writing a call option results in a payoff pattern similar to that of a short position in the underlying stock.

(Module 31.2, LOS 31.c)

Related Material

SchweserNotes - Book 4

15. (A) **less.**

Explanation

The delta of an at-the-money call option is typically close to 0.5. The delta of a long position in the underlying stock is 1.0 by definition.

(Module 31.7, LOS 31.k)

Related Material

SchweserNotes - Book 4

Susan Smith is analyzing the stock of FDL Inc. She has found the following quotations (all prices in dollars per share) on 91-day European options:

Exhibit 1

Option Strike Price	Call Premium	Put Premium
50	5.28	1.54
55	2.64	3.33
60	1.14	X
Other Information		
Risk-free interest rates	6%	
Annual volatility	30%	
FDL Inc share price	$53	

*FDL Inc currently does not pay dividends

Smith wants to value the equity call options of FDL Inc. using the Black-Scholes-Merton (BSM) model. She wants to understand the assumptions and the limitations of the model and asks David Wang for help. Wang provides the information shown in Exhibits 2 and 3.

Exhibit 2

Assumptions of BSM Model
Assumption 1
The underlying asset returns follow a normal distribution
Assumption 1
Options are European style

Exhibit 3

Assumptions of BSM Model	Implications
The risk-free rate is known and constant over the options life	**Implication 1** Useful for pricing options on bond prices and interest rates
The continuously compounded yield on the asset is constant	**Implication 2** BSM model can be modified to account for cash flows on the underlying.

16. (C) $7.27.

Explanation

The put-call parity equation is: $C + X / (1 + r)^t = P + S$

Hence $P = C — S + X/ (1 + r)^t = 1.14 — 53 + 60 / (1.06)^{91/365} = \$7.27$

(Module 31.1, LOS 31.a)

Related Material

SchweserNotes - Book 4

17. (A) Write a call option, buy a put option, buy one share, borrow the PV of strike.

Explanation

The $55-strike put is currently underpriced compared to its synthetic

($3.33 < $3.85).

We therefore will want to go long the underpriced put and short the overpriced synthetic to make an arbitrage profit.

Susan will be obliged to sell the shares at $55, so she also needs to buy the stock (so that she has it to deliver) and borrow the PV of the $55 strike ($55 / $(1.06)^{91/365}$ = $54.21). The net result will be a receipt of $2.64 – $3.33 – $53 + 54.21 = $0.52 per share. Thus:

Write a call option, buy a put option, buy one share, and borrow the PV of strike.

(Module 31.1, LOS 31.a)

Related Material

SchweserNotes - Book 4

18. (B) **Only one of the two assumptions is correct.**

Explanation

The assumptions used by the basic BSM model include:

- The underlying asset price follows a geometric Brownian motion process. The continuously compounded return is normally distributed. Under this framework, change in asset price is continuous: there are no abrupt jumps.
- The (continuously compounded) risk-free rate is constant and known. Borrowing and lending are both at the risk-free rate.
- The volatility of the returns on the underlying asset is constant and known.
- Markets are "frictionless." There are no taxes, no transactions costs, and no restrictions on short sales or the use of short-sale proceeds. Continuous trading is possible, and there are no arbitrage opportunities in the marketplace.
- The (continuously compounded) yield on the underlying asset is constant.
- The options are European options (i.e., they can only be exercised at expiration).

Assumption 1 is incorrect. Asset prices are lognormally distributed and asset returns are normally distributed.

Assumption 2 is correct.

(Module 31.6, LOS 31.f)

Related Material

SchweserNotes - Book 4

19. (C) **Only one of the two implications is correct.**

Explanation

Implication 1 is incorrect. The BSM model assumes a constant risk-free interest rate but interest rate volatility is a key factor that determines the value of options on bonds and interest rate related contracts. Hence BSM is not useful for pricing options on bond prices and interest based derivatives.

Implication 2 is correct. BSM allows for constant continuously compounded divided yield (i.e., cash flow on the underlying) by adjusting the asset value by the present value of the expected cash flows.

(Module 31.6, LOS 31.f)

Related Material

SchweserNotes - Book 4

20. (B) **sometimes worthwhile to exercise puts early but not calls.**

Explanation

After early exercise of a put, and in particular a deep in-the-money put, the sale proceeds can be invested at the risk-free rate and may earn interest worth more than the time value of the put option. The same is not true for call options: early exercise of call options on non-dividend-paying stock is never optimal.

(Module 31.6, LOS 31.i)

Related Material

SchweserNotes - Book 4

21. (B) **at the money.**

Explanation

When the option is at the money, changes in volatility will have the greatest effect on the option value.

(Module 31.7, LOS 31.k)

Related Material

SchweserNotes - Book 4

22. (B) **Volatility.**

Explanation

A decrease/increase in the volatility of the price of the underlying asset will decrease/increase both put values and call values. A change in the values of the other inputs will have opposite effects on the values of puts and calls.

(Module 31.7, LOS 31.k)

Related Material

SchweserNotes - Book 4

23. (A) **A portfolio of put options on an interest rate.**

Explanation

A long floor (floor buyer) has the same general expiration-date payoff diagram as that for long interest rate put position.

(Module 31.6, LOS 31.j)

Related Material

SchweserNotes - Book 4

24. (C) **writing a series of puts on fixed income securities.**

Explanation

For a bondholder, a cap, which puts a maximum on floating rate interest payments, is equivalent to writing a series of puts on fixed income securities. These would require the buyer to pay when rates rise and bond prices fall, negating interest rate increases above the cap rate. Writing a series of interest rate calls, not puts, would be an equivalent strategy. Calls on fixed income securities would pay when rates decrease, not when they increase.

(Module 31.6, LOS 31.j)

Related Material

SchweserNotes - Book 4

25. (B) futures is sometimes worthwhile but never is for options on forwards.

Explanation

Early exercise of in-the-money American options on futures is sometimes worthwhile because the immediate mark to market upon exercise will generate funds that can earn interest. It is never worthwhile for options on forwards because no funds are generated until the settlement date of the forward contract.

(Module 31.3, LOS 31.b)

Related Material

SchweserNotes - Book 4

26. (C) put option.

Explanation

After exercising a deep in-the-money put early, the sale proceeds can be invested at the risk-free rate, and in this way the investor may earn interest worth more than the time value of the put. Non-dividend-paying call options on stock will never be exercised early because the minimum price of the option always exceeds its exercise value. European options cannot be exercised early.

(Module 31.1, LOS 31.d)

Related Material

SchweserNotes - Book 4

You are interested in derivative products, particularly with a view to identifying arbitrage opportunities. You start with bond futures:

- The cheapest to deliver (CTD) bond underlying the T-bond futures contract maturing in five months is a 4.6% T-Bond currently priced at $1,002.33 (full price) per $1,000 par. The CTD paid its last coupon four months ago, and its conversion factor is 1.13. The risk free rate is 2.99%.

Peter Wang, one of your colleague, knew of your interest in derivative products advises you to consider interest rate options and swaptions. Wang makes the following comments:

Comment 1:	An investor having a long position in a call option on a bond has the same position as if he is long an interest rate floor.
Comment 2:	A borrower of a floating rate loan can create an interest rate collar by buying an interest rate cap and selling an interest rate floor and the cap sets the maximum interest rate payable by the borrower.
Comment 3:	A payer swaption is the right to enter into a specific swap at some date in the future as the fixed-rate payer. A payer swaption becomes more valuable if an equivalent swap at the market rate is higher than the strike rate.

27. (A) $867.20.

Explanation

The no-arbitrage price for T-Bond futures is given by the formula:

$$QFP = \{(\text{full price}) \times (1+R_f)^T - AI_T - FVC)(1/CF)$$

The full price of the bond = clean price + accrued interest. Since the bond pays semi-annual coupons, and four months have passed since the last coupon, there are two months until the next coupon.

Accrued interest (AI) = (days since last coupon / days between coupons) x $ semiannual coupon. In this question we have not been told days but instead have months.

AI_T in the formulae represents the accrued interest at the maturity of the futures contract. Given the last coupon was 4 months ago the next coupon of $23 will be in two months' time. At the maturity of the futures contract in five months we will be 3 months through the coupon period, hence:

AI_T = (3 months / 6 months) x $23 = $11.5

The next coupon will be in two months' time (four months' ago, plus six months) and will equal $23. FVC in the above formula is this coupon compounded up to the futures maturity, three months later, so FVC = $23 x 1.02993 / 12 = $23.17.

Thus, QFP = [($1,002.33 x 1.02995 / 12) — $11.5 — $23.17] / 1.13 = $867.20

(Module 30.3, LOS 30.d)

Related Material

SchweserNotes - Book 4

28. (A) correct.

Explanation

For a long call option on a bond, when interest rates decrease, bond prices rise hence call value increases. Similarly, for an interest rate put, when interest rate decreases, the long put value increases.

(Module 31.6, LOS 31.j)

Related Material

SchweserNotes - Book 4

29. (A) correct.

Explanation

Interest rate cap pays when the reference rate exceeds the strike rate. As such, the borrower can use the payment from the interest rate cap to offset the higher interest payment on the floating rate loan. Hence the strike rate on the cap is the maximum interest rate that the borrower has to pay. By selling the floor, the premium received on the floor will help to offset some of the premium paid on the cap. In addition, the collar also hedged the interest rate exposure of the loan through the strike rate of the cap and floor.

(Module 31.6, LOS 31.j)

Related Material

SchweserNotes - Book 4

30. (A) correct.

Explanation

Comment 3 is a correct description of a payer swaption.

(Module 31.6, LOS 31.j)

Related Material

SchweserNotes - Book 4

31. (B) low5er value in all cases.

Explanation

An expected dividend during the term of an option will decrease the value of a call option.

(Module 31.6, LOS 31.h)

Related Material

SchweserNotes - Book 4

32. (C) underlying asset has positive cash flows.

Explanation

Positive cash flows in the form of dividends will lower the price of the stock making it closer to being "in the money" which increases the value of the option as the stock price gets closer to the strike price.

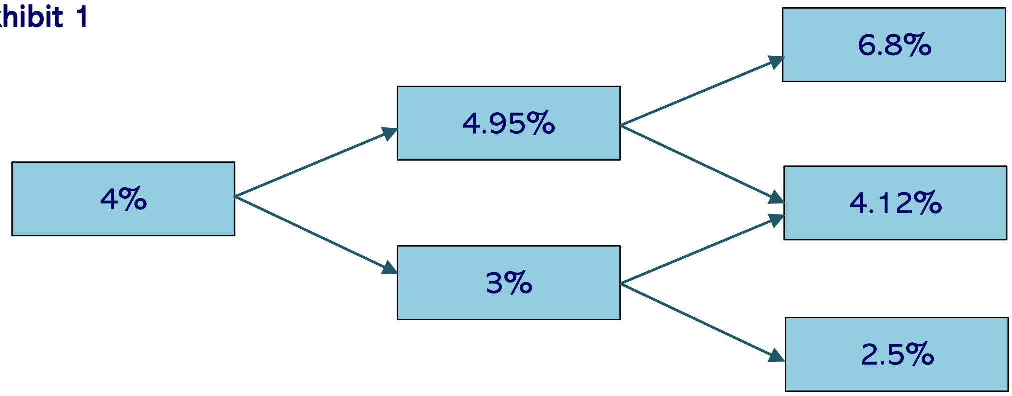
(Module 31.7, LOS 31.k)

Related Material

SchweserNotes - Book 4

Lowell Wood is using the binomial option-pricing model to price interest rate options. She has obtained the following 2-year, annual rate tree (based on an assumed volatility of interest rates of 25%).

Exhibit 1



Wood has been asked help a colleague with the valuation of an interest rate put. The interest rate put option has 2 years to maturity and a strike price of 4.5% and is based on 360 day MRR. The option has a notional principal of $10m.

Wood has discovered that the Black model may be used to price options on interest rates by viewing the interest rate option as an option on a FRA. She is currently writing a research note for her team and makes the following three notes regarding the Black model:

Note 1:	"When using the Black model care needs to be taken to ensure that the payoff is discounted from the end of the borrowing and lending period (i.e., the maturity of the rate underlying the FRA), rather than the exercise date of the option."
Note 2:	"Given an interest rate option is an option on a FRA, call options will gain in value when interest rates rise and put options will fall in value."
Note 3:	"The accrual period needs to be factored in when valuing the option. This is because quoted rates are annual rates but in reality, the time between the FRA expiration and the maturity of borrowing and lending may not be one year. The accrual period can be viewed as a fraction of a year."

Wood asks for information about interest rate caps and floors. Newman makes the following comments:

Comment 1:	"A long FRA can be viewed as equivalent to a long interest rate call and a short interest rate put with the same strike and time to expiration."
Comment 2:	"Given a cap is a series of interest rate call options with identical strike prices and a floor is a series of interest rate put options also with identical strike prices, a short cap and long floor with identical strike prices would create a pay fixed receive floating interest rate swap."

33. (B) $64,250.

Explanation

Step 1:

At the expiry of the option at T_2 consider whether the option will be exercised in each of the forward scenarios.

Remember an interest rate put option allows the holder the right but not obligation to pay floating and receive fixed. The strike price (in this case 4.5%) is the fixed rate in an interest rate option. The put will be exercised in the forward rate is less than the fixed rate.

Forward rate 6.8% — option lapses

Forward rate 4.12% — option exercised

Forward rate 2.5% — option exercised

Step 2:

We now calculate the pay off on the option at the options expiry given each interest rate scenario. This assumes the payoff on the option is at T_2, which is technically incorrect as in reality the payoff is at the end of the borrowing and lending period (T_3) rather than the expiry of the option (T_2).

If we exercise the interest rate option we then enter pay floating receive fixed until the end of the borrowing and lending period.

Calculate the payoff on the put at T_2 in each scenario:

(interest rate received — interest rate paid) x days / 360 x notional principal

forward rate 6.8% — option lapses payoff zero

forward rate 4.12% = (0.045 – 0.0412) x 360 / 360 x $10m = $38,000

forward rate 2.5% = (0.045 – 0.025) x 360 / 360 x $10m = $200,000

Step 3:

Discount the payoffs back through the binomial interest rate tree to T_0. Note that in a binomial interest rate tree we always have a 50% chance of an up move and a 50% chance of a down move. Discount the probability weighted amounts from T_2 back to T_1 at the relevant forward rate.

Value at T_1 upper node:

$$\left[(\$0 + \$38,000)\,\tfrac{1}{2}\right] / 1.0495 = \$18,103.86$$

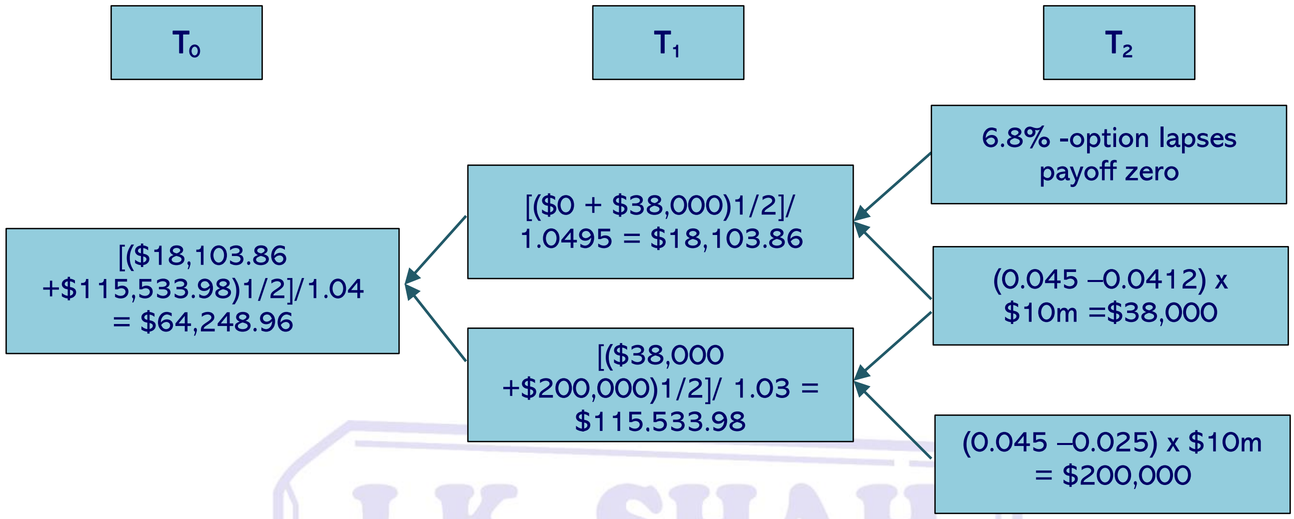
Value at T_1 lower node:

$$\left[(\$38,000 + \$200,000)\,\tfrac{1}{2}\right] / 1.03 = \$115,533.98$$

Discount the probability weighted amounts back from T1 to T0 at the 1 period spot rate to arrive that the price (premium) on the interest rate put.

$$\left[\left(\$18{,}103.86+\$115{,}533.98\right)\tfrac{1}{2}\right]/1.04=\$64{,}248.96$$

Diagrammatically:



(Module 31.5, LOS 31.e)

Related Material

SchweserNotes - Book 4

34. (A) All three notes are correct.

Explanation

Note 1: Correct

The payoff is not at the expiration of the option and FRA. Once exercised into the FRA the payoff occurs at the end of the pay fixed receive floating period (FRA maturity).

Note 2: Correct

Long FRA represents pay fixed and receive floating from the expiry of the FRA until the end of the borrowing and lending period. Long FRAs will increase in value as interest rates rise as the forward rate on a new FRA with the same expiry and borrowing and lending period will be higher. Given the underlying in an interest rate option is a long FRA the call option will increase in value as the FRA gains value. As with all options the call will increase in value as the underlying increases.

Note 3: Correct

In the Black model used to value interest rate option the FRA rate and strike price are annualized rates. Given the time between FRA expiry and the end of borrowing and lending is unlikely to be a year the present value of the payoff will need to be unannualized.

(Module 31.6, LOS 31.j)

Related Material

SchweserNotes - Book 4

35. (A) correct.

Explanation

Newman has replicated a long FRA position (pay fixed receive floating). Long call, short put with same strike and expiry = Long FRA

Short call, long put with same strike and expiry = Short FRA

(Module 31.6, LOS 31.j)

Related Material

SchweserNotes - Book 4

36. (C) incorrect as long floor, short cap would create a pay floating, receive fixed interest rate swap.

Explanation

Pay fixed, receive floating swap (payer swap) can be created by going long an interest rate cap and short an interest rate put.

A cap comprises of caplets where each caplet is in effect a call option on a FRA. A floor represents a series of floorlets where each floorlet is an interest rate put option.

Long floor and short cap with identical strike prices will create receiver swap (i.e., pay floating and receive fixed).

(Module 31.6, LOS 31.j)

Related Material

SchweserNotes - Book 4

37. (C) buy a payer swaption.

Explanation

A payer swaption will give Roberts the right to pay a fixed rate below market if rates rise.

(Module 31.6, LOS 31.j)

Related Material

SchweserNotes - Book 4

38. (C) smaller (negative) number.

Explanation

For call options larger gamma means that as the asset price increases, the delta of option A increases more than the delta of option B. Since the number of calls to hedge is (–1/delta)x(number of shares), the number of calls necessary for the hedge is a smaller (negative) number for option A than for option B.

(Module 31.7, LOS 31.l)

Related Material

SchweserNotes - Book 4

Nathan Detroit, a speculator, has come to you for technical advice regarding the pricing of swaps. He hopes to make big money in the swaps market from the exploitation of pricing discrepancies, but lacks an understanding of the principles underlying the pricing of swaps.

He asks you to consider a two-year, fixed-for-fixed, currency swap with semiannual payments. The domestic currency is the U.S. dollar and the foreign currency is the U.K. pound. The current exchange rate is $1.60 per pound. You forecast that the exchange rate would be $1.41 on the first settlement date. The notional principal of the swap is set at $10 million. The USD and £ term structure are shown in Exhibits 1 and 2 below.

Exhibit 1: Current USD Term Structure

Number of Days	MRR	Present Value Factor
180	0.0585	0.9716
360	0.0605	0.9430
540	0.0596	0.9179
720	0.0665	0.8826

Exhibit 2: Current £ Term Structure

Number of Days	MRR	Present Value Factor
180	0.0493	0.9759
360	0.0450	0.9569
540	0.0519	0.9278
720	0.0551	0.9007

Detroit has heard about the European put-call parity theorem and believes a synthetic call can be created through the use of a European put with the same strike as the call, a zero coupon bond with a face value equal to the strike price of the put and an underlying asset relating to the put and the call.

Detroit makes two comments regarding the BSM model and the Black model as follows:

Comment 1:	$N(d_1)$ in the BSM is the probability that the option will expire in the money.
Comment 2:	The probability that a receiver swaption will expire in the money is $N(-d_2)$.

39. (C) 6.32%.

Explanation

For a swap with n settlement periods, the semi-annual fixed payment per $1 of notional principal is calculated as:

$$\frac{1 - \text{PV factor for last cash flow}}{\sum_{i=1 \text{ to } n} \text{PV factor for ith cash flow}} = \frac{1-0.8826}{0.9716+0.9430+0.9180+0.8826} = 0.0316$$

The annualized fixed payment per $1 of notional principal is therefore:

$$0.0316 \times \frac{360}{180} = 0.0632 \text{ or } 6.32\%$$

(Module 30.6, LOS 30.e)

Related Material

SchweserNotes - Book 4

40. (A) £165,000.

Explanation

The semi-annual fixed payment per pound of notional principal is calculated as:

$$\frac{1 - \text{PV factor for last cash flow}}{\sum_{i=1 \text{ to } n} \text{PV factor for ith cash flow}} = \frac{1-0.9007}{0.9759+0.9519+0.9278+0.9057} = 0.0264$$

Given the dollar notional principal of $10 million, the notional principal in British pounds (using the exchange rate on the date the swap is initiated) will be:

$$\frac{\$10 \text{ million}}{\$1.60} = \$6.25\text{m}$$

Hence the payment made every 6 months will be £6.25m x 0.0264 = £165,000.

(Module 30.6, LOS 30.f)

Related Material

SchweserNotes - Book 4

41. (A) Buy the put and the underlying.

Explanation

When dealing with questions based on synthetics the solution can be arrived at by using put call parity.

Recall put + share = call + present value of bond PV(X)

The European put-call parity states that the payoffs of a fiduciary call (long zero coupon bond and long call) are equivalent to the payoffs of a protective put (long underlying and long put).

So a synthetic call = put + stock — PV(X)

We can then ignore the value of the pure discount bond PV(X) because its value is unaffected by the stock's price, leaving us with put + stock.

(Module 31.1, LOS 31.a)

Related Material

SchweserNotes - Book 4

42. (B) Only one statement is correct.

Explanation

$N(d_2)$ in both BSM and the Black model is the probability of the underlying asset being greater than the strike price at expiry. In the case of a swaption this would be the rate on a payer swap at expiry of the swaption being greater than the strike price (fixed rate) of the payer swaption.

$N(d_1)$ is a little more complicated than $N(d_2)$. It adjusts the current value of the underlying asset to represent the expected value of the underlying at expiry ignoring any values less that the strike price and factor in the probability of exercise. In practice this makes $N(d_1)$, a conditional probability, slightly bigger than $N(d_2)$ and unconditional probability. For the exam you should focus on knowing that $N(d_2)$ is the probability of exercise and that $N(d_1)$ is slightly higher. Do not waste too much time trying to understand exactly what $N(d_1)$ is doing.

If the $N(d_2)$ is the probability that ST > X at expiry (i.e., a call option is exercised) then 1 — $N(d_2)$ must be the probability that ST < X at expiry (i.e., the probability that a put option is exercised). It is common to write $N(—d_2)$ rather than 1 — (Nd_2).

(Module 31.6, LOS 31.j)

Related Material

SchweserNotes - Book 4

43. (A) possible by purchasing 28 shares and writing 100 calls.

Explanation

S^+ = 28(1.15) = 32.20; S^- = 28(0.87) = 24.36.

C^+ = 32.20 — 30 = 2.20, C^- = 0.

H = (2.20 — 0) / (32.20 — 24.36) = 0.281

$$C_0 = hS_0 + \frac{(-hS^+ + C^+)}{(1 + R_f)}$$

= 0.281 x 28 + [(- 0.281 x 32.20 + 2.20) / 1.03] = 1.22

Since the call price of $2.07 > 1.22, an arbitrage profit can be earned by writing calls and purchasing 0.281 shares per call written.

(Module 31.4, LOS 31.c)

Related Material

SchweserNotes - Book 4

44. (C) by solving the Black-Scholes model for the volatility using market values for the stock price, exercise price, interest rate, time until expiration, and option price.

Explanation

Implied volatility is found by "backing out" the volatility estimate using the current option price and all other values in the Black-Scholes model.

(Module 31.7, LOS 31.n)

Related Material

SchweserNotes - Book 4

45. (B) put and a short call on MRR with a strike rate of 8% and twelve months to expiration.

Explanation

Interest rate swaps can be replicated with a series of put and call positions with expiration dates on the payment dates of the swap. For a receiver swap (where we pay floating and receive fixed), we need an option position that pays when floating rates fall and that requires a payment to be made when rates increase. A long interest rate put plus a short interest rate call would accomplish this. The strike rate of the options corresponds to the fixed rate of the FRA. The expiration of the option coincides with the MRR determination date.

(Module 31.6, LOS 31.j)

Related Material

SchweserNotes - Book 4

46. (C) long position in a put option on the stock.

Explanation

The combined payoff pattern of a short position in a stock and a long call option on the stock is the same as the payoff pattern of a long put option on the stock.

(Module 31.2, LOS 31.c)

Related Material

SchweserNotes - Book 4

47. (B) incorrect about use of $e^{-rT}XN(-d_2)$ borrowed funds.

Explanation

Dilla is correct about calls being similar to a leveraged investment in $N(d_1)$ units of stock but is incorrect about the quantity of borrowed funds. It should be $e^{-rT}XN(-d_2)$.

(Module 31.6, LOS 31.g)

Related Material

SchweserNotes - Book 4

48. (B) possible by purchasing 100 calls and short selling 57 shares.

Explanation

S^+ = 55(1.20) = 66; S^- = 55 (0.85) = 46.75. C^+ = 66 – 55 = 11, C^- = 0.

H = (11 – 0) / (66 – 46.75) = 0.5714

$$C_0 = hS_0 + \frac{(-hS^+ + C^+)}{(1+R_f)}$$

= (0.571 x 55) + [((-0.571 x 66) + 11) / 1.05] = 5.99

Since the call price of $4.92 < $5.99, an arbitrage profit can be earned by buying calls and short selling 0.571 shares per call bought.

(Module 31.4, LOS 31.c)

Related Material

SchweserNotes - Book 4

49. (C) –0.75.

Explanation

The call option delta is:

$$delta_{call} = \frac{\$0.50}{\$2.00} = 0.25$$

The put option delta is 0.25 – 1 = –0.75.

(Module 31.7, LOS 31.l)

Related Material

SchweserNotes - Book 4

50. (B) an increasing function of the underlying asset's volatility.

Explanation

Since an option has limited risk but significant upside potential, its value always increases when the volatility of the underlying asset increases.

(Module 31.7, LOS 31.k)

Related Material

SchweserNotes - Book 4

51. (A) **owning a series of calls on fixed income securities.**

Explanation

A floor, which puts a minimum on floating rate interest payments is equivalent to owning calls on fixed income securities which will pay when interest rates fall. Owning interest rate puts, rather than writing them, would be equivalent to the floor. Puts on fixed income securities pay when interest rates increase.

(Module 31.6, LOS 31.j)

Related Material

SchweserNotes - Book 4

52. (C) **owning a series of interest rate calls.**

Explanation

The issuer of the note is borrowing at a floating rate, and will have higher interest expenses if rates increase. A cap is equivalent to owning a series of interest rate calls at the cap rate that will pay the difference between the market rate and the cap rate. If interest rates increase, the payoff from the calls will compensate the borrower for the higher interest expenses.

(Module 31.6, LOS 31.j)

Related Material

SchweserNotes - Book 4

53. (C) **an obligation to enter a swap in the future as the fixed-rate payer.**

Explanation

A receiver swaption gives its owner the right to receive fixed, the writer has an obligation to pay fixed.

(Module 31.6, LOS 31.j)

Related Material

SchweserNotes - Book 4

54. (C) **increase as the volatility of the underlying asset increases because call options have limited risk but unlimited upside potential.**

Explanation

A higher volatility makes it more likely that options end up in the money and can be exercised profitably, while the down side risk is strictly limited to the option premium.

(Module 31.7, LOS 31.n)

Related Material

SchweserNotes - Book 4

55. (B) **exercise price.**

Explanation

The value of a call option decreases as the exercise price increases.

(Module 31.7, LOS 31.k)

Related Material

SchweserNotes - Book 4

56. (B) **a delta hedge will perform more poorly over time.**

Explanation

Gamma measures the rate of change of delta (a high gamma could mean that delta will be higher or lower) as the asset price changes and, graphically, is the curvature of the option price as a function of the stock price. Delta measures the slope of the function at a point. The greater gamma is (the more delta changes as the asset price changes), the worse a delta hedge will perform over time.

(Module 31.7, LOS 31.m)

Related Material

SchweserNotes - Book 4

57. (C) **Theta.**

Explanation

Theta describes the change in option price in response to the passage of time. Since option holders would prefer that value not decay too quickly, an option with a low theta value is desirable.

(Module 31.7, LOS 31.k)

Related Material

SchweserNotes - Book 4

58. (C) **Buy a European put option; buy the same stock; sell a European call option.**

Explanation

According to put-call parity we can write a riskless pure-discount bond position as:

$$X/(1+R_f)^T = P_0 + S_0 - C_0$$

We can then read off the right-hand side of the equation to create a synthetic position in the riskless pure-discount bond. We would need to buy the European put, buy the same underlying, stock, and sell the European call

(Module 31.1, LOS 31.a)

Related Material

SchweserNotes - Book 4

59. (A) Buy the stock; buy a European put option on the same stock with the same exercise price and the same maturity; short an amount equal to the present value of the exercise price worth of a pure-discount riskless bond.

Explanation

According to put-call parity we can write a European call as:

$$C_0 = P_0 + S_0 - X/(1+R_f)^T$$

We can then read off the right-hand side of the equation to create a synthetic position in the call.

We would need to buy the European put, buy the stock, and short or issue a riskless pure-discount bond equal in value to the present value of the exercise price.

(Module 31.1, LOS 31.a)

Related Material

SchweserNotes - Book 4

Frank Potter, CFA, a financial adviser for Star Financial, LLC has been hired by John Williamson, a recently retired executive from Reston Industries. Over the years Williamson has accumulated $10 million worth of Reston stock and another $2 million in a cash savings account. Potter has a number of unconventional investment strategies for Williamson's portfolio; many of the strategies include the use of various equity derivatives.

Potter's first recommendation involves the use of a total return equity swap. Potter outlines the characteristics of the swap in Table 1. In addition to the equity swap, Potter explains to Williamson that there are numerous options available for him to obtain almost any risk return profile he might need. Potter suggest that Williamson consider options on both Reston stock and the S&P 500. Potter collects the information needed to evaluate options for each security. These results are presented in Table 2.

Table 1: Specification of Equity Swap

Term	3 years
Notional principal	$10 million
Settlement frequency	Annual, commencing at end of year 1
Fairfax pays to broker	Total return on Reston Industries stock
Broker pays to Fairfax	Total return on S&P 500 Stock Index

Table 2: Option Characteristics

	Reston	S&P 500
Stock price	$50.00	$1,400.00
Strike price	$50.00	$1,400.00
Interest rate	6.00%	6.00%
Dividend yield	0.00%	0.00%
Time to expiration (years)	0.5	0.5
Volatility	40.00%	17.00%
Beta Coefficient	1.23	1
Correlation	0.4	

Table 3: Regular and Exotic Options (Option Values)

	Reston	S&P 500
European call	$6.31	$6.31
European put	$4.83	$4.83
American call	$6.28	$6.28
American put	$4.96	$4.96

Table 4: Reston Stock Option Sensitivities

	Delta
European call	0.5977
European put	–0.4023
American call	0.5973
American put	–0.4258

Table 5: S&P 500 Option Sensitivities

	Delta
European call	0.622
European put	–0.378
American call	0.621
American put	–0.441

Potter has also been asked to evaluate the interest rate risk of an intermediate size bank. The bank has a large floating rate liability of $100,000,000 on which it pays the MRR on a quarterly basis. Potter is concerned about the significant interest rate risk the bank incurs because of this liability: since most of the bank's assets are invested in fixed rate instruments there is a considerable duration mismatch. Some of the bank's assets are floating rate notes tied to MRR, however, the total par value of these securities is significantly less than the liability position.

Potter considers both swaps and interest rate options. The interest rate options are 2-year caps and floors with quarterly exercise dates. Potter wishes to hedge the entire liability.

Potter has obtained the prices for an at-the-money 6 month cap and floor with quarterly exercise. These are shown in Table 6.

Table 6: At-the-Money 0.5 year Cap and Floor Values

Price of at-the-money Cap	$133,377
Price of at-the-money Floor	$258,510

60. (B) Buy 497,141 put options.

Explanation

Number of put options $= \left(\text{Reston Portfolio Value / Stock Price}_{\text{Reston}}\right) / \text{-DeltaPut}$

Number of put options = ($10,000,000 / $50.00) / -0.4023 = -497,141 meaning buy 497,141 put options.)

(Module 31.1, LOS 31.l)

Related Material

SchweserNotes - Book 4

61. (B) $0.

Explanation

Swaps are priced so that their value at inception is zero.

(Module 30.8, LOS 30.g)

Related Material

SchweserNotes - Book 4

62. (A) Buy an interest rate cap.

Explanation

Buying a cap, combined with a floating rate liability, limits the exposure to interest rate increases (i.e., no exposure to interest rate increases above strike rate). The floating rate borrower will still benefit from interest rate decreases.

(Module 31.6, LOS 31.j)

Related Material

SchweserNotes - Book 4

63. (C) Buy a floor.

Explanation

Buying a floor combined with a floating rate assets limits the exposure to interest rate decreases (i.e., no exposure to interest rate decreases below strike rate) while the floating rate holder is still able to benefit from interest rate increases. Ideally, Potter should consider matching the bank's asset position against the bank's liability position.

(Module 31.6, LOS 31.j)

Related Material

SchweserNotes - Book 4

64. (B) $9.13.

Explanation

First, calculate the probability of an up move or a down move:

P_u = (1 + 0.08 – 0.833) / (1.20 – 0.833) = 0.673

P_d = 1 – 0.673 = 0.327

Two up moves produce a stock price of 40 x 1.44 = 57.60 and a call value at the end of two periods of 20.60. An up and a down move leave the stock price unchanged at 40 and produce a call value of 3. Two down moves result in the option being out of the money. The value of the call option is discounted back one year and then discounted back again to today. The calculations are as follows:

C^+ = [20.6(0.673) + 3(0.327)] / 1.08 = 13.745

C^- = [3(0.673) + 0 (0.327)] / 1.08 = 1.869

Call value today = [13.745(0.673) + 1.869(0.327)] / 1.08 = 9.13

(Module 31.1, LOS 31.b)

Related Material

SchweserNotes - Book 4

65. (C) -$3,600.

Explanation

At expiration, the fixed rate is 3.763% which is below the exercise rate of 3.8%. The purchaser of the receiver swaption will exercise the option which allows them to receive a fixed rate of 3.8% from the writer of the option and pay the current rate of 3.763%.

The equivalent of two payments of (0.038 - 0.03763) x (180/360) x (10,000,000) will be made to the receiver swaption. One payment would have been received in 6 months and will be discounted back to the present at the 6-month rate. One payment would have been received in 12 months and will be discounted back to the present at the 12-month rate.

The first payment, discounted to the present is (0.038 - 0.03763) x (180/360) x (10,000,000) x (1/1.018) = $1,817.28.

The second payment, discounted to the present is (0.038 - 0.03763) x (180/360) x (10,000,000) x (1/1.038) = $1,782.27

The total payoff for the writer is -$3,599.55.

(Module 31.6, LOS 31.j)

Related Material

SchweserNotes - Book 4

66. (A) **delta as the price of the underlying security changes.**

Explanation

Gamma is the rate of change in delta. It measures how fast the price sensitivity changes as the underlying asset price changes.

(Module 31.7, LOS 31.k)

Related Material

SchweserNotes - Book 4

67. (C) **call and a short put on MRR with a strike rate of 6% and two years to expiration.**

Explanation

A long FRA is replicated by a long IR call and short IR put with expiration corresponding to the FRA settlement date.

(Module 31.6, LOS 31.j)

Related Material

SchweserNotes - Book 4

68. (C) **Buy a European call option; short a European put option; invest the present value of the exercise price in a riskless pure-discount bond.**

Explanation

According to put-call parity we can write a stock position as:

$$S_0 = C_0 - P_0 + X / (1 + R_f)^T$$

We can then read off the right-hand side of the equation to create a synthetic position in the stock. We would need to buy the European call, sell the European put, and invest the present value of the exercise price in a riskless pure-discount bond.

(Module 31.1, LOS 31.a)

Related Material

SchweserNotes - Book 4

Lucy Wang is the Chief Financial Officer of Sam Corporation. Sam Corporation has floating rate liabilities and wants to hedge against the possibility of rising interest rates. Wang is looking into using swaptions to hedge against interest rate risk.

The board of Sam Corporation are not familiar with both swaps and swaptions. To explain the characteristics of swaps, Wang explains to the board that swaps are similar to interest rate options.

Wang decides to use a swaption to hedge against interest rate risk. She knows that there are two types of swaptions just like there are both call and put options, and that the cash flows on swaps can be replicated using swaptions.

Lucy is very interested in the application of the Black model in pricing swaptions. After a quick search online she has found the following:

$$\text{pay} = (AP)PVA[SFR \times N(d_1) - X \times N(d_2)] \times NP$$

where:

pay = value of the payer swaption

AP = 1 / number of settlement periods per year in the underlying swap

X = exercise rate specified in the swaption

NP = notional principal

Lucy is confused regarding what the notation PVA stands for and states:

"There are multiple payoffs on a swaption, each being the difference between the market swap rate at expiration and the exercise rate at each settlement date, over the swaps life. Given each payoff arises at a different point in time over the swaps life, each must be discounted back to the current period using discount rate specific to when it occurs. The PVA therefore must be an annuity factor summing all these specific discount rates."

Lucy also states:

"A payer swaption is right to enter a swap with a fixed rate equal to the strike price at the options maturity. The payoffs will therefore represent the difference in the swaptions strike price and the fixed market swap rate at the time of option expiry. Given we are comparing the payer swap underlying the swaption, versus the market rate of a payer swap at the option maturity, the floating payments can be ignored as they will offset. This is why the Black model only includes values for the current market swap fixed rate and fixed rate of the swap underlying the swaption (i.e., the strike)."

69. (C) **The swap can be replicated by buying a package of interest rate call options and selling a package of interest rate put options at the same strikes.**

Explanation

The pay-fixed party in an interest rate swap receives a net payment if the floating rate is above the fixed rate and pays when the floating rate is below the fixed rate. This payoff characteristic is similar to buying a package of interest rate call options (receive payment when reference rate is above the strike rate) and selling a package of interest rate put options (pays when the reference rate is below the strike rate) at the same strikes.

(Module 31.6, LOS 31.j)

Related Material

SchweserNotes - Book 4

70. (B) **The swap can be replicated by buying a payer swaption and selling a receiver swaption at the same strike.**

Explanation

The swap can be replicated by buying a payer swaption and selling a receiver swaption at the same strikes.

Payer swaption is the right to enter the swap as the fixed-rate payer and receiver swaption is the right to enter as fixed-rate receiver. Payer swaption increases in value when the floating rate increases, similar to that for the pay-fixed party in an interest rate swap. Receiver swaption increases in value when the floating rate decreases. To replicate the negative value suffered by the pay-fixed interest rate party when floating rate decreases, the swap can be replicated by selling a receiver swaption as the negative value increases when rates fall.

(Module 31.6, LOS 31.j)

Related Material

SchweserNotes - Book 4

71. (A) **Lucy is correct about the definition of the PVA and the offsetting of floating rates.**

Explanation

Wang is correct about both parts.

Exercise of a payer swaption results in a swap with fixed payments equal to the strike price on the swaption and the receipt of floating payments.

The payoff on a swaption is the difference between the fixed swap rate that can be achieved by exercising the option and the market fixed rate on a swap at expiry. This payoff will be received at every settlement date over the swaps life. The PVA is therefore an annuity factor, which will discount all the payoffs back to the current period.

The market swap at expiry of the swaption is pay fixed at the market rate and receive floating payments. The swaption can be exercised into a pay fixed (at swaption strike) and receive floating swap. The floating payments are identical on the two swaps and therefore can be ignored for valuation purposes.

(Module 31.6, LOS 31.j)

Related Material

SchweserNotes - Book 4

72. (C) **A payer swaption will only be exercised in the market swap fixed rate at expiry is greater than the swaption's strike price.**

Explanation

SFR in the Black formula represents the current market swap fixed rate not the fixed rate at expiration. Both the Black and BSM formulas use the current price of the underlying asset.

The Black formula, like BSM computes the expected value of what you will receive less the present value of what you will pay given assumptions such as volatility.

$N(d_2)$ = only considers probability that the option is exercised (i.e., the probability of the market SFR being greater than the strike at expiration). $N(d_1)$ has to adjust the current SFR to the expected SFR at expiration given that the option will only be exercised if the market SFR > strike rate. The result is that in the Black and BSM formulas that $N(d_1)$ is slightly higher than $N(d_2)$.

A payer swaption will only be exercised if the strike rate is lower than the market rate on a pay fixed swap at expiry. When a swaption is exercised the holder will pay fixed at the strike price and receive floating. This will be attractive when the fixed rate on a pay fixed swap in the market is higher. The floating payments can be ignored as the rate on swaption when exercised and the rate on a new swap at expiry would be identical.

(Module 31.6, LOS 31.j)

Related Material

SchweserNotes - Book 4

73. (C) **The yield on the underlying has a known and constant volatility.**

Explanation

Assumptions of BSM include:

- The volatility of the return on the underlying is known and constant.
- If the underlying instrument pays a yield, it is expressed as a continuous known and constant yield at an annualized rate.

(Module 31.6, LOS 31.f)

Related Material

SchweserNotes - Book 4

74. (A) $0.57.

Explanation

Two down moves produce a stock price of 38 x 0.87^2 = 28.73 and a put value at the end of two periods of 6.27. An up and a down move, as well as two up moves leave the put option out of the money. You are directly given the probability of up = 0.68. Hence, the down probability = 0.32. The value of the put option is [0.32^2 x 6.27] / 1.06^2 = $0.57.

(Module 31.1, LOS 31.b)

Related Material

SchweserNotes - Book 4

75. (C) correct.

Explanation

Jenkins is correct about both probabilities.

(Module 31.6, LOS 31.g)

Related Material

SchweserNotes - Book 4