Reading 4

PROBABILITY TREES AND CONDITIONAL EXPECTATIONS

1. (A) $3.29.

Explanation

State of the Economy (Unconditional Probability)	Conditional Probability	Joint Probability	EPS	Joint Probability × EPS
GOOD 60%	70%	60% × 70% = 42%	$5.00	42% × $5.00 = $2.10
	30%	60% × 30% = 18%	$3.50	18% × $3.50 = $0.63
BAD 40%	80%	40% × 80% = 32%	$1.50	32% × $1.50 = $0.48
	20%	40% × 20% = 8%	$1.00	8% × $1.00 = $0.08
Expected EPS = ∑ Joint Probability × EPS				**$3.29**

(Module 4.1, LOS 4.a)

2. (C) conditional expectation

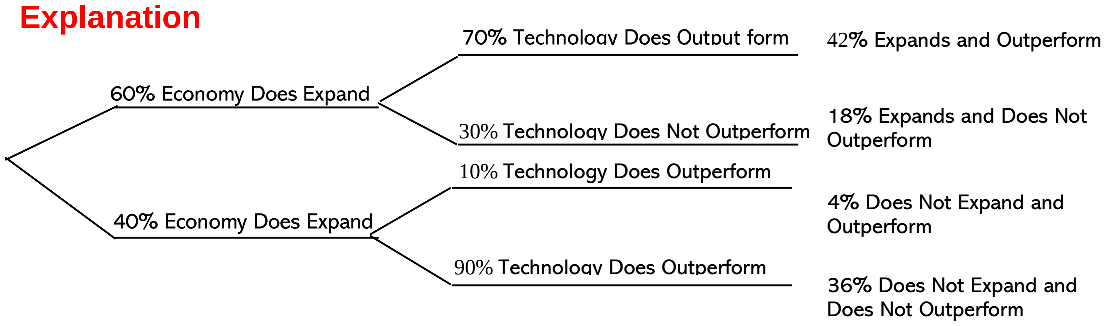
Explanation

This is a conditional expectation. The analyst indicates how an expected value will change given another event.

(Module 4.1, LOS 4.b)

3. (C) 67%.

Explanation



Using the new information, we can use Bayes' formula to update the probability.

P (economy does not expand | tech does not outperform) = P (economy does not expand and tech does not outperform) / P (tech does not outperform).

P (economy does not expand and tech does not outperform)

= P (tech does not outperform. | economy does not expand) × P (economy does not expand)

= 0.90 × 0.40 = 0.36.

P (economy does expand and tech does not outperform)

= P(tech does not outperform |economy does expand) × P(economy does expand)

= 0.30 × 0.60 = 0.18.

P(economy does not expand)

= 1.00 – P(economy does expand)

= 1.00 – 0.60 = 0.40.

P(tech does not outperform | economy does not expand)

= 1.00 – P(tech does outperform | economy does not expand)

= 1.00 – 0.10 = 0.90.

P(tech does not outperform)

= P(tech does not outperform and economy does not expand) + P(tech does not outperform and economy does expand)

= 0.36 + 0.18 = 0.54.

P(economy does not expand | tech does not outperform)

= P(economy does not expand and tech does not outperform) / P(tech does not outperform)

= 0.36 / 0.54 = 0.67.

(Module 4.1, LOS 4.c)

4. (B) 0.625.

Explanation

Using the total probability rule, we can compute the

P(B): $P(B) = [P(B \mid A) \times P(A)] + [P(B \mid A^C) \times P(A^C)]$

$P(B) = [0.5 \times 0.4] + [0.2 \times 0.6] = 0.32$

Using Bayes' formula, we can solve for

P(A | B): $P(A \mid B) = [\, P(B \mid A) \div P(B)\,] \times P(A) = [0.5 \div 0.32] \times 0.4 = 0.625$

(Module 4.1, LOS 4.c)

5. (B) 16.5%

Explanation

E(r) = (0.70 × 0.12) + (0.30 × 0.40 × 0.30) + (0.30 × 0.60 × 0.25) = 0.165.

(Module 4.1, LOS 4.a)

6. (A) $2.47

Explanation

We need to calculate of probability weighted average payoff.

Since the probability of the coin landing on its edge is 0.02, the probability of each of the other two events is 0.49.

The expected payoff is:

(0.02 × $50) + (0.49 × $1) + (0.49 × $2) = $2.47.

Outcome	Probability	Payoff	Probability × Payoff
Edge	2 / 100 = 2%	$50	2% × $50
Heads	49%	$1	49% × $1
Tails	49%	$2	49% × $2
Expected Payoff = ∑ Probability × Payoff			**$2.47**

(Module 4.1, LOS 4.a)

7. (A) 37%.

Explanation

Given a set of prior probabilities for an event of interest, Bayes' formula is used to update the probability of the event, in this case that the car we already know has a radio is red. Bayes' formula says to divide the Probability of New Information given Event by the Unconditional Probability of New Information and multiply that result by the Prior Probability of the Event. In this case, P(red car has a radio) = 0.70 is divided by 0.76 (which is the Unconditional Probability of a car having a radio (40% are red of which 70% have radios) plus (60% are blue of which 80% have radios) or ((0.40) × (0.70)) + ((0.60) × (0.80)) = 0.76.) This result is then multiplied by the Prior Probability of a car being red, 0.40. The result is (0.70 / 0.76) × (0.40) = 0.37 or 37%.

(Module 4.1, LOS 4.c)

8. (C) 69%.

Explanation

Given a set of prior probabilities for an event of interest, Bayes' formula is used to update the probability of the event, in this case that the company we have already selected will experience a decline in earnings next year. Bayes' formula says to divide the Probability of New Information given Event by the Unconditional Probability of New Information and multiply that result by the Prior Probability of the Event. In this case, P(company having a decline in earnings next year) = 0.20 is divided by 0.26 (which is the Unconditional Probability that a company having an earnings decline will have a negative ratio (90% have negative ratios of the 20% which have earnings declines) plus (10% have negative ratios of the 80% which do not have earnings declines) or ((0.90) × (0.20)) + ((0.10) × (0.80)) = 0.26.)This result is then multiplied by the Prior Probability of the ratio being negative, 0.90. The result is (0.20 / 0.26) × (0.90) = 0.69 or 69%.

(Module 4.1, LOS 4.c)

9. (A) 0.211.

Explanation

According to Bayes' formula: P (B | default) = P(default and B) / P(default).

P (default and B) = P (default | B) × P(B) = 0.250 × 0.300 = 0.075

P (default and CCC) = P (default | CCC) × P(CCC) = 0.400 × 0.700 = 0.280

P(default) = P(default and B) + P(default and CCC) = 0.355

P (B | default) = P (default and B) / P(default) = 0.075 / 0.355 = 0.211

(Module 4.1, LOS 4.c)

10. (C) 15.0%; 7.58%.

Explanation

Mean = (0.4) (10) + (0.4) (12.5) + (0.2) (30) = 15%

Var = $(0.4)(10 - 15)^2 + (0.4)(12.5 - 15)^2 + (0.2)(30 - 15)^2 = 57.5$

Standard deviation = $\sqrt{57.5} = 7.58$

(Module 4.1, LOS 4.a)

11. (C) 11.55%.

Explanation

The expected return on Portfolio A is a probability-weighted average of 17%, 14%, 12% and 8%.

Expected return = (0.15) (0.17) + (0.20)(0.14) + (0.25)(0.12) + (0.40)(0.08)

= 0.1155 or 11.55%.

Scenario	Probability	Return on Portfolio A	Portfolio × Weight
A	15%	17%	15 × 17%
B	20%	14%	20% × 14%
C	25%	12%	25% × 12%
D	40%	8%	40% × 8%
Probability Weighted Average Return ∑ **Probability × Weight**			**11.55%**

(Module 4.1, LOS 4.a)

12. (C) $57.00.

Explanation

The expected value if the overall market decreases is 0.4($60) + (1 – 0.4) ($55) $57.

(Module 4.1, LOS 4.b)

13. (C) 0.75.

Explanation

Using the information of the stock being good, the probability is updated to a conditional probability:

P(John | good) = P(good and John) / P(good).

P(good and John) = P(good | John) × P(John) = 0.5 × 0.6 = 0.3.

P(good and Andrew) = 0.25 × 0.40 = 0.10.

P(good) = P(good and John) + P (good and Andrew) = 0.40.

P(John | good) = P(good and John) / P(good) = 0.3 / 0.4 = 0.75.

(Module 4.1, LOS 4.c)

14. (C) refining a forecast because of the occurrence of some other event.

Explanation

Conditional expected values are contingent upon the occurrence of some other event. The expectation changes as new information is revealed.

(Module 4.1, LOS 4.b)

15. (A) 1.7%.

Explanation

The standard deviation is the positive square root of the variance. The variance is the expected value of the squared deviations around the expected value, weighted by the probability of each observation.

The expected value is:
(0.5) × (0.12) + (0.3) × (0.1) + (0.2) × (0.15) = 0.12.

The variance is:
$(0.5) \times (0.12 - 0.12)^2 + (0.3) \times (0.1 - 0.12)^2 + (0.2) \times (0.15 - 0.12)^2 = 0.0003$.

The standard deviation is the square root of 0.0003 = 0.017 or 1.7%.
(Module 4.1, LOS 4.a)