

05

PORTFOLIO MATHEMATICS

1. For two random variables, $P(X = 20, Y = 0) = 0.4$, and $P(X = 30, Y = 50) = 0.6$. Given that $E(X)$ is 26 and $E(Y)$ is 30, the covariance of X and Y is:
- (A) 120.00.
 (B) 125.00.
 (C) 25.00
2. Personal Advisers, Inc., has determined four possible economic scenarios and has projected the portfolio returns for two portfolios for their client under each scenario. Personal's economist has estimated the probability of each scenario as shown in the table below. Given this information, what is the covariance of the returns on Portfolio A and Portfolio B?

Scenario	Probability	Return on Portfolio A	Return on Portfolio B
A	15%	18%	19%
B	20%	17%	18%
C	25%	11%	10%
D	40%	7%	9%

- (A) 0.002019.
 (B) 0.890223.
 (C) 0.001898.
3. The following table shows the weightings and expected returns for a portfolio of three stocks:

Stock	Weight	$E(R_x)$
V	0.40	12%
M	0.35	8%
S	0.25	5%

What is the expected return of this portfolio?

- (A) 9.05%.
 (B) 8.85%.
 (C) 8.33%.

4. For assets A and B we know the following: $E(R_A) = 0.10$, $E(R_B) = 0.10$, $\text{Var}(R_A) = 0.18$, $\text{Var}(R_B) = 0.36$ and the correlation of the returns is 0.6. What is the variance of the return of a portfolio that is equally invested in the two assets?
- (A) 0.1102.
(B) 0.1500.
(C) 0.2114.

5. The joint probability function for returns on an equity index (R_I) and returns on a stock (R_S) is given in the following table:

Return on stock (R_S)	Returns on Index (R_I)		
	$R_I = 0.16$	$R_I = 0.02$	$R_I = -0.10$
$R_S = 0.24$	0.25	0.00	0.00
$R_S = 0.03$	0.00	0.45	0.00
$R_S = -0.15$	0.00	0.00	0.30

Covariance between stock returns and index returns is closest to:

- (A) 0.019.
(B) 0.014.
(C) 0.029.
6. If Stock X has a standard deviation of returns of 18.9% and Stock Y has a standard deviation of returns equal to 14.73% and returns on the stocks are perfectly positively correlated, the standard deviation of an equally weighted portfolio of the two is:
- (A) 14.67%.
(B) 16.82%.
(C) 10.25%.

7. Given $P(X = 2) = 0.3$, $P(X = 3) = 0.4$, $P(X = 4) = 0.3$. What is the variance of X?
- (A) 0.3.
(B) 0.6.
(C) 3.0.

8. Use the following probability distribution.

State of the Economy	Probability	Return on Portfolio
Boom	0.30	15%
Bust	0.70	3%

The expected return for the portfolio is:

- (A) 9.0%.
(B) 6.6%.
(C) 8.1%.

9. The mean and standard deviation of returns for three portfolios are listed below in percentage terms.

Portfolio X: Mean 5%, standard deviation 3%.

Portfolio Y: Mean 14%, standard deviation 20%.

Portfolio Z: Mean 19%, standard deviation 28%.

Using Roy's safety-first criteria and a threshold of 4%, select the optimal portfolio.

- (A) Portfolio Y.
- (B) Portfolio X.
- (C) Portfolio Z.

10. For assets A and B we know the following: $E(R_A) = 0.10$, $E(R_B) = 0.20$, $\text{Var}(R_A) = 0.25$, $\text{Var}(R_B) = 0.36$ and the correlation of the returns is 0.6. What is the expected return of a portfolio that is equally invested in the two assets?

- (A) 0.3050.
- (B) 0.1500.
- (C) 0.2275.

11. Given the following probability distribution, find the covariance of the expected returns for stocks A and B.

Event	P(R _i)	R _A	R _B
Recession	0.10	-5%	4%
Below Average	0.30	-2%	8%
Normal	0.50	10%	10%
Boom	0.10	31%	12%

- (A) 17.4.
- (B) 10.9
- (C) 3.2

12. The following information is available concerning expected return and standard deviation of Pluto and Neptune Corporations:

	Expected Return	Standard Deviation
Pluto Corporation	11%	0.22
Neptune Corporation	9%	0.13

If the correlation between Pluto and Neptune is 0.25, determine the expected return and standard deviation of a portfolio that consists of 65% Pluto Corporation stock and 35% Neptune Corporation stock.

- (A) 10.3% expected return and 2.58% standard deviation.
- (B) 10.3% expected return and 16.05% standard deviation.
- (C) 10.0% expected return and 16.05% standard deviation.

13. Assume two stocks are perfectly negatively correlated. Stock A has a standard deviation of 10.2% and stock B has a standard deviation of 13.9%. What is the standard deviation of the portfolio if 75% is invested in A and 25% in B?
- (A) 0.00%.
(B) 0.17%.
(C) 4.18%.
14. The mean and standard deviation of returns on three portfolios are listed below in percentage terms:
- Portfolio X: Mean 5%, standard deviation 3%.
 - Portfolio Y: Mean 14%, standard deviation 20%.
 - Portfolio Z: Mean 19%, standard deviation 28%.
- Using Roy's safety-first criteria and a threshold of 3%, which of these is the optimal portfolio
- (A) Portfolio Z.
(B) Portfolio X.
(C) Portfolio Y.
15. An investor has two stocks, Stock R and Stock S in her portfolio. Given the following information on the two stocks, the portfolio's standard deviation is closest to:
- $\sigma_R = 34\%$
 - $\sigma_S = 16\%$
 - $r_{R,S} = 0.67$
 - $W_R = 80\%$
 - $W_S = 20\%$
- (A) 29.4%.
(B) 7.8%.
(C) 8.7%.
16. For two random variables, $P(X = 2, Y = 10) = 0.3$, $P(X = 6, Y = 2.5) = 0.4$, and $P(X = 10, Y = 0) = 0.3$. Given that $E(X)$ is 6 and $E(Y)$ is 4, the covariance of X and Y is:
- (A) -12.0.
(B) 24.0.
(C) 6.0.

17. An investor is considering investing in one of the following three portfolios:

Statistical Measures	Portfolio X	Portfolio Y	Portfolio Z
Expected annual return	12%	17%	22%
Standard deviation of return	14%	20%	25%

If the investor's minimum acceptable return is 5%, the optimal portfolio using Roy's safetyfirst criterion is:

- (A) Portfolio X.
(B) Portfolio Y.
(C) Portfolio Z.
18. Use the following probability distribution to calculate the standard deviation for the Portfolio.

State of the Economy	Probability	Return on Portfolio
Boom	0.30	15%
Bust	0.70	3%

- (A) 5.5%.
(B) 6.0%.
(C) 6.5%.
19. Given the following probability distribution, find the standard deviation of expected returns:

Event	P(R _A)	R _A
Recession	0.10	-5%
Below Average	0.30	-2%
Normal	0.50	10%
Boom	0.10	31%

- (A) 10.04%.
(B) 12.45%.
(C) 7.00%.
20. Expected returns and standard deviations of returns for three portfolios are shown in the following table:

Portfolio	Expected Return	Standard Deviation
1	9%	5%
2	8%	4%
3	7%	3%

Assuming the risk-free rate is 3%, an investor who wants to minimize the probability of returns less than 5% should choose:

- (A) Portfolio 2.
- (B) Portfolio 3.
- (C) Portfolio 1.

21. Tully Advisers, Inc., has determined four possible economic scenarios and has projected the portfolio returns for two portfolios for their client under each scenario. Tully's economist has estimated the probability of each scenario, as shown in the table below. Given this information, what is the standard deviation of returns on portfolio A?

Scenario	Probability	Return on Portfolio A	Return on Portfolio B
A	15%	18%	19%
B	20%	17%	18%
C	25%	11%	10%
D	40%	7%	9%

- (A) 1.140%.
- (B) 5.992%.
- (C) 4.53%.

22. Three portfolios with normally distributed returns are available to an investor who wants to minimize the probability that the portfolio return will be less than 5%. The risk and return characteristics of these portfolios are shown in the following table:

Portfolio	Expected return	Standard deviation
Epps	6%	4%
Flake	7%	9%
Grant	10%	15%

Based on Roy's safety-first criterion, which portfolio should the investor select?

- (A) Flake.
- (B) Grant.
- (C) Epps.

23. Which of the following portfolios provides the best "safety first" ratio if the minimum acceptable return is 6%

Portfolio	Expected Return (%)	Standard Deviation (%)
1	13	5
2	11	3
3	9	2

- (A) 1.
- (B) 3.
- (C) 2.

24. The covariance of the returns on investments X and Y is 18.17. The standard deviation of returns on X is 7%, and the standard deviation of returns on Y is 4%. What is the value of the correlation coefficient for returns on investments X and Y?
- (A) +0.65.
(B) +0.32.
(C) +0.85.
25. If given the standard deviations of the returns of two assets and the correlation between the two assets, which of the following would an analyst least likely be able to derive from these?
- (A) Strength of the linear relationship between the two.
(B) Expected returns.
(C) Covariance between the returns.
26. The covariance of returns on two investments over a 10-year period is 0.009. If the variance of returns for investment A is 0.020 and the variance of returns for investment B is 0.033, what is the correlation coefficient for the returns?
- (A) 0.444.
(B) 0.350.
(C) 0.687.
27. The returns on assets C and D are strongly correlated with a correlation coefficient of 0.80. The variance of returns on C is 0.0009, and the variance of returns on D is 0.0036. What is the covariance of returns on C and D?
- (A) 0.00144.
(B) 0.03020.
(C) 0.40110.
28. Compute the standard deviation of a two-stock portfolio if stock A (40% weight) has a variance of 0.0015, stock B (60% weight) has a variance of 0.0021, and the correlation coefficient for the two stocks is -0.35 ?
- (A) 1.39%.
(B) 0.07%.
(C) 2.64%.

29. Which of the following portfolios provides the optimal "safety first" return if the minimum acceptable return is 9%?

Portfolio	Expected Return (%)	Standard Deviation (%)
1	13	5
2	11	3
3	9	2

- (A) 1.
- (B) 2.
- (C) 3.

30. What is the standard deviation of a portfolio if you invest 30% in stock one (standard deviation of 4.6%) and 70% in stock two (standard deviation of 7.8%) if the correlation coefficient for the two stocks is 0.45?

- (A) 6.20%.
- (B) 0.38%.
- (C) 6.83%

31. Joe Mayer, CFA, projects that XYZ Company's return on equity varies with the state of the economy in the following way:

State of Economy	Probability of Occurrence	Company Returns
Good	.20	20%
Normal	.50	15%
Poor	.30	10%

The standard deviation of XYZ's expected return on equity is closest to:

- (A) 3.5%.
- (B) 12.3%.
- (C) 1.5%.

