Reading 3

STATISTICAL MEASURES OF ASSET RETURNS

1. (B) 7.7%.

Explanation

Geometric mean = $[(1.10)\ (1.14)\ (1.12)\ (1.10)\ (0.90)\ (1.12)]^{1/6} - 1 = 0.0766$, or 7.66%

(Module3.1, LOS 3.a)

2. (B) 19.24.

Explanation

Calculations are as follows:

1. Sample mean = (125 + 175 + 150 + 155 + 135) / 5 = 148
2. Sample Variance = $[(125 - 148)^2 + (175 - 148)^2 + (150 - 148)^2 + (155 - 148)^2 + (135 - 148)^2] / (5 - 1) = 1{,}480 / 4 = 370$
3. Sample Standard Deviation = $370^{1/2} = 19.24\%$.

(Module 3.1, LOS 3.b)

3. (C) Y.

Explanation

The coefficient of variation, CV = standard deviation / arithmetic mean, is a common measure of relative dispersion (risk). $CV_X = 0.7 / 0.9 = 0.78$; $CV_Y = 4.7 / 1.2 = 3.92$; and $CV_Z = 5.2 / 1.5 = 3.47$. Because a higher CV means higher relative risk, Security Y has the highest relative risk.

(Module 3.1, LOS 3.b)

4. (B) 12.

Explanation

Calculate the mean:

$$\frac{25+15+35+45+55}{5} = 35$$

To get the mean absolute deviation, sum the deviations around the mean (ignoring the sign), and divide by the number of observations:

$$\frac{10+20+0+10+20}{5} = 12$$

(Module 3.1, LOS 3.b)

5. (C) 4.96% less than

Explanation

The geometric return is calculated as follows:

$[(1 + 0.25) (1 + 0.15) (1 + 0.12) (1 - 0.08) (1 – 0.14)]^{1/5} – 1$,

or $[1.25 \times 1.15 \times 1.12 \times 0.92 \times 0.86]^{0.2} – 1 = 0.4960$, or **4.96%.**

The geometric return will always be less than or equal to the arithmetic return. In this case the arithmetic return was 6%.

(Module 3.1, LOS 3.a)

6. (A) 7.08%.

Explanation

Compound annual growth rate is the geometric mean.

$(1.056 \times 1.2267 \times 0.9477)^{1/3} – 1 = 7.08\%$

(Module 3.1, LOS 3.a)

7. (A) 7.0%.

Explanation

(0.333)(0.06) + (0.333)(0.10) + 0.333(0.05) = 0.07

(Module 3.1, LOS 3.a)

8. (C) 23%.

Explanation

The third quartile is calculated as: $L_y = (7 + 1) (75/100) = 6$. When we order the observations in ascending order: 7%, 10%, 12%, 15%, 20%, 23%, 27%, "23%" is the sixth observation from the left.

(Module 3.1, LOS 3.a)

9. (A) higher probability of extreme upside returns and higher chance of extreme downside returns.

Explanation

A leptokurtic distribution (a distribution with kurtosis measures greater than 3) is more peaked in the middle (data more clustered around the mean) and has fatter tails at the extremes (greater probability of outliers).

(Module 3.2, LOS 3.c)

10. (C) 8.0.

Explanation

The sample variance is found by taking the sum of all squared deviations from the mean and dividing by (n – 1).

$[(15 – 3)^2 + (2 – 3)^2 + (5 – 3)^2 + (-7 – 3)2 + (0 – 3)^2] / (5 – 1) = 64.5$

The sample standard deviation is found by taking the square root of the sample variance. $\sqrt{64.5} = 8.03$

(Module 3.1, LOS 3.b)

11. (B) the median is greater than the mean.

Explanation

For a distribution with negative skewness, mean < median < mode.

(Module 3.2, LOS 3.c)

12. (A) Mean, median, mode.

Explanation

In a negatively skewed distribution, the mean is less than the median, which is less than the mode.

(Module 3.2, LOS 3.c)

13. (A) 21.0(%²)

Explanation

$[(4 - 5)^2 + (10 - 5)^2 + (1 - 5)2] / (3 - 1) = 21(\%^2)$.

(Module 3.1, LOS 3.b)

14. (B) –0.5%.

Explanation

$G = [(1.10)(0.85)(1.00)(1.05)]^{0.25} - 1$

$G = (0.98175)^{0.25} - 1 = 0.9954 - 1 = -0.00459 \approx -0.5\%$

Note: Taking a number to the 0.25 power is the same as taking the fourth root of the number.

(Module 3.1, LOS 3.a)

15. (A) positively skewed.

Explanation

The distance to the left from the mode to the beginning of the range is 8. The distance to the right from the mode to the end of the range is 15. Therefore, the distribution is skewed to the right, which means that it is positively skewed.

(Module 3.2, LOS 3.c)

16 (B) 0.97%.

Explanation

The sample standard deviation equals the square root of the sum of the squares of the position returns less the mean return, divided by the number of observations in the sample minus one.

Position	Return (%)	(Return – Mean)^2
A	1.3	0.60
B	1.4	0.46
C	2.2	0.02
D	3.4	1.76
Mean	8.3/4 = 2.075	Sum = 2.83
Std. Dev. = $[2.83 / (4 – 1)]^{0.5}$ = 0.97		

(Module 3.1, LOS 3.b)

17 (A) 9.1%.

Explanation

$(1.104 \times 1.081 \times 1.032 \times 1.15)^{0.25} – 1 = 9.1\%$

(Module 3.1, LOS 3.a)

18. (C) 20%; 3%.

Explanation

(14 + 20 + 24 + 22) / 4 = 20 (mean)

Take the absolute value of the differences and divide by n:

MAD = [|14 – 20| + |20 – 20| + |24 – 20| + |22 – 20|] / 4 = 3%.

(Module 3.1, LOS 3.b)

19. (A) 0.5% to 5.2%.

Explanation

The interquartile range is from the first quartile (25th percentile) to the third quartile (75th percentile) and is represented as the box in a box-and-whisker plot. The horizontal line within the box represents the median (50th percentile).

(Module 3.1, LOS 3.a)

20. (A) 0.167.

Explanation

The coefficient of variation is the standard deviation divided by the mean:

5 / 30 = 0.167.

(Module 3.1, LOS 3.b)

21. (B) median.

Explanation

Median = middle of distribution = 8 (middle number);

Mean = (3 + 3 + 5 + 8 + 9 + 13 + 17) / 7 = 8.28;

Mode = most frequent observation = 3.

(Module 3.1, LOS 3.a)

22. (B) 2.75%; 3.00%.

Explanation

Geometric Mean:

$(1.15 \times 1.02 \times 1.05 \times 0.93 \times 1.0)^{1/5} - 1 = 1.1454^{\,1/5} - 1 = 2.75\%$

Arithmetic Mean: (15% + 2% + 5% – 7% + 0%) / 5 = 3.00%

(Module 3.1, LOS 3.a)

23. (A) has positive excess kurtosis.

Explanation

A distribution that has a greater percentage of small deviations from the mean and a greater percentage of large deviations from the mean will be leptokurtic and will exhibit positive excess kurtosis. The distribution will be taller (more peaked) with fatter tails than a normal distribution.

(Module 3.2, LOS 3.c)

24. (C) mean > median > mode.

Explanation

When a distribution is positively skewed the right-side tail is longer than normal due to outliers. The mean will exceed the median, and the median will generally exceed the mode because large outliers falling to the far-right side of the distribution can dramatically influence the mean.

(Module 3.2, LOS 3.c)

25. (B) 17.0%.

Explanation

With 9 observations, the location of the 70th percentile is (9 + 1) (70 / 100) = 7. The seventh observation in ascending order is 17.0%.

(Module 3.1, LOS 3.a)

26. (A) describes the degree to which a distribution is not symmetric about its mean.

Explanation

The degree to which a distribution is not symmetric about its mean is measured by skewness. Excess kurtosis which is measured relative to a normal distribution, indicates the peakedness of a distribution, and also reflects the probability of extreme outcomes.

(Module 3.2, LOS 3.c)

27. (A) The geometric mean may be used to estimate the average return over a oneperiod time horizon because it is the average of one-period returns.

Explanation

The arithmetic mean may be used to estimate the average return over a one-period time horizon because it is the average of one-period returns. Both remaining statements are true.

(Module 3.1, LOS 3.a)

28. (C) 141.7.

Explanation

The formula for determining quantiles is: $L_y = (n + 1)(y) / (100)$. Here, we are looking for the seventh decile (70% of the observations lie below) and the formula is: (21)(70) / (100) =14.7. The seventh decile falls between 141.0 and 142.0, the fourteenth and fifteenth numbers from the left. Since L is not a whole number, we interpolate as:

141.0 + (0.70) (142.0 – 141.0) = 141.7.

(Module 3.1, LOS 3.a)

29. (C) 0.78.

Explanation

The coefficient of variation expresses how much dispersion exists relative to the mean of a distribution. It is a measure of risk per unit of mean return.

CV = s / mean. 3.56 / 4.56 = 0.781, or 78%.

(Module 3.1, LOS 3.b)

30. (A) 20%.

Explanation

Coefficient of variation, CV = standard deviation / mean. The standard deviation is the square root of the variance, or $4^{½} = 2$. So, CV = 2 / 10 = 20%.

(Module 3.1, LOS 3.b)

31. (A) more peaked than a normal distribution.

Explanation

A distribution with positive excess kurtosis is more peaked and has fatter tails than a normal distribution.

(Module 3.2, LOS 3.c)

32. (B) 18.05%.

Explanation

Find the weighted mean of the returns.

(0.10 × 0.02) + (0.30 × 0.095) + (0.60 × 0.25) =18.05%

Asset	Weight	Return	Weight × Return
Cash	10%	2%	10% × 2% = 0.2%
Bonds	30%	9.5%	30% × 9.5% = 2.85%
Stock	60%	25%	60% × 25% = 15%
Weighted Average Return **∑ Weight × Probability**			**18.05%**

(Module 3.1, LOS 3.a)

33. (A) 15.8%.

Explanation

Here we need to multiply the returns by the proportion that each stock represents in the portfolio then sum.

Stock	Return	Invested	Proportion of Portfolio	Return × Proportion
P	20%	\$7,000	7/12	20% × 7/12
Q	10%	\$5,000	5/12	10% × 5/12
Total		**\$12,000**		**15.83%**

(Module 3.1, LOS 3.a)

34. (B) 4.49%.

Explanation

The geometric return is calculated as follows:

$[(1 + 0.20) \times (1 + 0.15) \times (1 + 0.0)\ (1 - 0.05)\ (1 - 0.05)]^{1/5} - 1$,

or $[1.20 \times 1.15 \times 1.0 \times 0.95 \times 0.95]^{0.2} - 1 = 0.449$, or **4.49%.**

(Module 3.1, LOS 3.a)

35. (C) 8.75%; 8.34%.

Explanation

$(14 + 6 + (-5) + 20) / 4 = 8.75$.

$((1.14 \times 1.06 \times 0.95 \times 1.20)^{0.25} - 1 = 8.34\%$.

(Module 3.1, LOS 3.a)

36. (C) A leptokurtic distribution has fatter tails than a normal distribution.

Explanation

A leptokurtic distribution is more peaked than normal and has fatter tails. However, the excess kurtosis is greater than zero.

(Module 3.2, LOS 3.c)

37. (C) 13.1%; 13.7%.

Explanation

The median is the midpoint of the data points. In this case there are 13 data points and the midpoint is the 7th term.

The formula for determining quantiles is: Ly = (n + 1)(y) / (100). Here, we are looking for the third quintile (60% of the observations lie below) and the formula is: (14)(60) / (100) = 8.4. The third quintile falls between 13.6% and 13.9%, the 8th and 9th numbers from the left Since L is not a whole number, we interpolate as: 0.136 + (0.40) (0.139 – 0.136) = 0.1372, or13.7%.

(Module 3.1, LOS 3.a)

38. (C) The harmonic mean.

Explanation

$$\text{Harmonic mean} \frac{4}{\frac{1}{1.04}+\frac{1}{1.03}+\frac{1}{1.02}+\frac{1}{1.30}} - 1 = 0.0864 = 8.64\%$$

$$\text{Geometric mean} = \left[(1.04)(1.03)(1.02)(1.30)\right]^{\frac{1}{4}} - 1 = 0.0917 = 9.17\%$$

$$\text{Arthmetic mean} = \frac{4\% + 3\% + 2\% + 30\%}{4} = 9.75\%$$

(Module 3.1, LOS 3.a)

39. (C) 9.1%.

Explanation

Standard deviation

$= [\sum i\left(xi - \overline{X}\right)^2 / (n-1)]^{1/2}$

$= (744.10 / 9)^{1/2} = 9.1\%.$

(Module 3.1, LOS 3.b)

40. (C) positive skewness has a long-left tail.

Explanation

A distribution with positive skewness has long right tails.

(Module 3.2, LOS 3.c)

41. (B) highest and lowest 2.5% of observations.

Explanation

A 5% trimmed means discards the highest 2.5% and lowest 2.5% of observations and is the arithmetic average of the remaining 95% of observations.

(Module 3.1, LOS 3.a)

42. (B) 11.00; 10.97.

Explanation

Arithmetic Mean: 12 + 14 + 9 + 13 + 7 = 55; 55 / 5 = 11

Geometric Mean: $[(1.12 \times 1.14 \times 1.09 \times 1.13 \times 1.07)^{1/5}] - 1 = 10.97\%$

(Module 3.1, LOS 3.a)

43. (A) have a mean that is less than its median.

Explanation

With the low outlier included, the distribution will be negatively skewed. For a negatively skewed distribution, the mean is less than the median, which is less than the mode.

(Module 3.2, LOS 3.c)

44. (B) the two variables have a negative linear association.

Explanation

A correlation coefficient of –0.74 suggests a relatively strong negative linear association between the two variables. We cannot interpret the correlation coefficient directly as a measure of the probability that the two variables will change in opposite directions.

(Module 3.2, LOS 3.d)

45. (A) Winsor zed mean.

Explanation

A winsorized mean is a technique for removing the distorting effects of outliers by replacing them with less extreme values. The arithmetic and geometric means are based on all observations and therefore include the impact of outliers.

(Module 3.1, LOS 3.a)

46. (B) Mode, median, mean.

Explanation

In a positively skewed distribution, the mode is less than the median, which is less than the mean.

(Module 3.2, LOS 3.c)

47. (A) mean only.

Explanation

Mean is affected because it is the sum of all values / number of observations. Median is not affected as it the midpoint between the top half of values and the bottom half of values.

(Module 3.1, LOS 3.a)

48. (A) It has a lower percentage of small deviations from the mean than a normal distribution.

Explanation

A distribution with positive excess kurtosis has a higher percentage of small deviations from the mean than normal. So it is more "peaked" than a normal distribution. A distribution with positive skew has a mean > mode.

(Module 3.2, LOS 3.c)

49. (B) 60%.

Explanation

The coefficient of variation expresses how much dispersion exists relative to the mean of a distribution and is found by CV = s / mean, or 0.25 / 0.42 = 0.595, or 60%.

(Module 3.1, LOS 3.b)

50. (B) 65% of all the observations are below that observation.

Explanation

If the observation falls at the sixty-fifth percentile, 65% of all the observations fall below that observation.

(Module 3.1, LOS 3.a)

51. (B) 4.40.

Explanation

The CV = the standard deviation of returns / mean return

= 8.8% / 2.0% = 4.4.

The CV is a measure of risk per unit of mean return. When ranking portfolios based on the CV, a lower value is preferred to higher.

(Module 3.1, LOS 3.b)

52. (C) 5.6%.

Explanation

The mean absolute deviation is found by taking the mean of the absolute values of deviations from the mean.

(|15 – 3| + |2 – 3| + |5 – 3| + |-7 – 3| + |0 – 3|) / 5 = 5.60%

(Module 3.1, LOS 3.b)

53. (B) leptokurtic.

Explanation

A distribution that is more peaked than normal is leptokurtic. A leptokurtic distribution has fatter tails compared to a normal distribution. This means there is a greater chance of observing extreme outcomes. Market returns are leptokurtic. A distribution that is flatter than a normal distribution is termed platykurtic.

(Module 3.2, LOS 3.c)

54. (A) 1.80.

Explanation

The coefficient of variation is equal to the standard deviation of returns divided by the mean return.

Mean return = (17.0% + 12.2% + 3.9% – 8.4%) / 4 = 6.175%

Year	Return	$(R – 6.175\%)^2$
1	17.0%	117.18
2	12.2%	36.30
3	3.9%	5.18
4	–8.4%	212.43
		Sum = 371.09

Sample standard deviation = $[371.09 / (4 – 1)]^{0.5}$ = 11.12%

Coefficient of variation = 11.12% / 6.175% = 1.80

(Module 3.1, LOS 3.b)

55. (B) **replaces outliers with less extreme returns.**

Explanation

The winsorized mean is a technique for dealing with outliers. For example, a 90% winsorized mean replaces the lowest 5% of values with the fifth percentile, and replaces the highest 5% of values with the 95th percentile. The arithmetic mean weights all observations equally. The geometric mean captures the compounded growth rate of the fund.

(Module 3.1, LOS 3.a)

56. (B) **lower than the arithmetic mean.**

Explanation

A trimmed mean discards a percentage of the highest and lowest observations, while a winsorized mean replaces a percentage of the highest and lowest observations with less extreme values. In this case the arithmetic mean would be influenced by the two highly positive returns, while a trimmed or winsorized mean would adjust for them and would likely be lower than the arithmetic mean.

(Module 3.1, LOS 3.a)

57. (A) not related linearly to economic growth.

Explanation

A correlation coefficient near zero indicates that two variables exhibit no linear relationship. This does not necessarily mean that the variables are unrelated because they might exhibit a nonlinear relationship.

(Module 3.2, LOS 3.d)

58. (A) Mean > median > mode.

Explanation

For the positively skewed distribution, the mode is less than the median, which is less than the mean.

(Module 3.2, LOS 3.c)

59. (A) 10.50%.

Explanation

Expected return is the weighted average of the individual expected values. The expected return is:

[(5,000) × (10.00) + (5,000) × (8.00) + (10,000) × (12.00)] / 20,000 = 10.50%.

(Module 3.1, LOS 3.a)

60. (B) the mode, but less than the mean.

Explanation

For a positively skewed distribution, the mean is greater than the median, and the median is greater than the mode. Their order reverses for a negatively skewed distribution.

(Module 3.2, LOS 3.c)

61. (B) The median is equal to the mode.

Explanation

The median is the mid-point or central number of returns arranged from highest to lowest or lowest to highest. In this case: 7, 8, 9, **12,** 12, 13, 14. The median return is 12%. The mode is the return that occurs most frequently. In this case, 12% is also the mode. The mean is 75 / 7 = 10.71%.

(Module 3.1, LOS 3.a)