Reading 5

PORTFOLIO MATHEMATICS

1. (A) 120.00.

Explanation

The covariance is COV(XY) = (0.4 × ((20 – 26) × (0 – 30))) + ((0.6 × (30 – 26) × (50 – 30))) = 120.

(Module 5.1, LOS 5.b)

2. (C) 0.001898.

Explanation

S	P(S)	Return on Portfolio A	$R_A - E(R_A)$	Return on Portfolio B	$R_B - E(R_B)$	$[R_A - E(R_A)] \times [R_B - E(R_B)] \times P(S)$
A	15%	18%	6.35%	19%	6.45%	0.000614
B	20%	17%	5.35%	18%	5.45%	0.000583
C	25%	11%	–0.65%	10%	–2.55%	0.000041
D	40%	7%	–4.65%	9%	–3.55%	0.000660
		$E(R_A)$ = 11.65%		$E(R_B)$ = 12.55%		$Cov(R_A, R_B)$ =0.001898

(Module 5.1, LOS 5.b)

3. (B) 8.85%.

Explanation

The expected return is simply a weighted average return.

Multiplying the weight of each asset by its expected return, then summing, produces:

E(RP)= 0.40(12) + 0.35(8) + 0.25(5) = 8.85%.

State of the Economy	Weight	$E(R_X)$	Probability × Return
V	0.40	12%	0.4 × 12%
M	0.35	8%	0.35 × 8%
S	0.25	5%	0.25 × 5%
Expected Return = Σ Weight × $E(R_X)$			**8.85%**

(Module 5.1, LOS 5.a)

4. (C) 0.2114.

Explanation

You are not given the covariance in this problem but instead you are given the correlation coefficient and the variances of assets A and B from which you can determine the covariance by Covariance = (correlation of A, B) × Standard Deviation of A) × (Standard Deviation of B).

Since it is an equally weighted portfolio, the solution is:

$[(0.5^2) \times 0.18] + [(0.5^2) \times 0.36] + [2 \times 0.5 \times 0.5 \times 0.6 \times (0.18^{0.5}) \times (0.36^{0.5})]$

$= 0.045 + 0.09 + 0.0764 = 0.2114$

(Module 5.1, LOS 5.a)

5. (B) 0.014.

Explanation

E(I) = (0.25 × 0.16) + (0.45 × 0.02) + (0.30 × –0.10) = 0.0190.

E(S) = (0.25 × 0.24) + (0.45 × 0.03) + (0.30 × –0.15) = 0.0285.

Covariance = [0.25 × (0.16 – 0.0190) × (0.24 – 0.0285)] + [0.45 × (0.02 – 0.0190) × (0.03 –0.0285)] + [0.30 × (–0.10 – 0.0190) × (–0.15 – 0.0285)] = 0.0138.

(Module 5.1, LOS 5.b)

6. (B) 16.82%.

Explanation

The standard deviation of two stocks that are perfectly positively correlated is the weighted average of the standard deviations:

0.5(18.9) + 0.5(14.73) = 16.82%.

This relationship is true only when the correlation is one. Otherwise, you must use the formula:

$\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\sigma_{1,2}}$

(Module 5.1, LOS 5.a)

7. (B) 0.6.

Explanation

The variance is the sum of the squared deviations from the expected value weighted by the probability of each outcome.

The expected value is E(X) = 0.3 × 2 + 0.4 × 3 + 0.3 × 4 = 3.

The variance is $0.3 \times (2 - 3)^2 + 0.4 \times (3 - 3)^2 + 0.3 \times (4 - 3)^2 = 0.6$.

(Module 5.1, LOS 5.a)

8. (B) 6.6%.

Explanation

The expected portfolio return is a probability-weighted average:

State of the Economy	Probability	Return on Portfolio	Probability × Return
Boom	0.30	15%	0.3 × 15% = 4.5%
Bust	0.70	3%	0.7 × 3% = 2.1%
Expected Return = ΣProbability × Return			**6.6%**

(Module 5.1, LOS 5.a)

9. (C) Portfolio Z.

Explanation

Portfolio Z has the largest value for the SFRatio: (19 – 4) / 28 = 0.5357.

For Portfolio X, the SFRatio is (5 – 4) / 3 = 0.3333.

For Portfolio Y, the SFRatio is (14 – 4) / 20 = 0.5000.

(Module 5.1, LOS 5.c)

10. (B) 0.1500.

Explanation

The expected return of a portfolio composed of n-assets is the weighted average of the expected returns of the assets in the portfolio:

$((w_1) \times (E(R_1)) + ((w_2) \times (E(R_2)) = (0.5 \times 0.1) + (0.5 \times 0.2) = 0.15$.

(Module 5.1, LOS 5.a)

11. (A) 17.4.

Explanation

Find the weighted average return for each stock.

Stock A: (0.10)(-5) + (0.30)(-2) + (0.50)(10) + (0.10)(31) = 7%.

Stock B: (0.10)(4) + (0.30)(8) + (0.50)(10) + (0.10)(12) = 9%.

Next, multiply the differences of the two stocks by each other, multiply by the probability of the event occurring, and sum. This is the covariance between the returns of the two stocks.

$[(-5 - 7) \times (4 - 9)](0.1) + [(-2 - 7) \times (8 - 9)](0.3) + [(10 - 7) \times (10 - 9)](0.5) + [(31 - 7) \times (12 - 9)](0.1) = 6.0 + 2.7 + 1.5 + 7.2 = 17.4$

(Module 5.1, LOS 5.b)

12. (B) 10.3% expected return and 16.05% standard deviation.

Explanation

$ER_{Port} = (W_{Pluto})(ER_{Pluto}) + (W_{Neptune})(ER_{Neptune})$

$= (0.65)(0.11) + (0.35)(0.09) = 10.3\%$

$\sigma p = [(w_1)^2(\sigma_1)^2 + (w_2)^2(\sigma_2)^2 + 2w_1w_2\sigma_1\sigma_2 r_{1,2}]^{1/2}$

$= [(0.65)^2 (22)^2 + (0.35)^2(13)^2 + 2(0.65)(0.35)(22)(13)(0.25)]^{1/2}$

$= [(0.4225)(484) + (0.1225)(169) + 2(0.65)(0.35)(22)(13)(0.25)]^{1/2}$

$= (257.725)^{1/2} = 16.0538\%$

(Module 5.1, LOS 5.a)

13. (C) 4.18%.

Explanation

The standard deviation of the portfolio is found by:

$[W_1^2\sigma_1^2+W_2^2\sigma_2^2 + 2W_1W_2\sigma_1\sigma_2r_{1,2}]^{0.5}$,

Or $[(0.75)^2(0.102)^2 + (0.25)^2(0.139)^2 + (2)(0.75)(0.25)(0.102)(0.139)(-1.0)]^{0.5}$

= 0.0418, or 4.18%.

(Module 5.1, LOS 5.a)

14. (B) Portfolio X.

Explanation

According to the safety-first criterion, the optimal portfolio is the one that has the largest value for the SFRatio (mean – threshold) / standard deviation.

For Portfolio X, (5 – 3) / 3 = 0.67.

For Portfolio Y, (14 – 3) / 20 = 0.55.

For Portfolio Z, (19 – 3) / 28 = 0.57.

(Module 5.1, LOS 5.c)

15. (A) 29.4%.

Explanation

The formula for the standard deviation of a 2-stock portfolio is:

s $= [W_A^2 s_A^2 + W_B^2 s_B^2 + 2W_A W_B s_A s_B r_{A,B}]^{1/2}$

s $= [(0.8^2 \times 0.34^2) + (0.2^2 \times 0.16^2) + (2 \times 0.8 \times 0.2 \times 0.34 \times 0.16 \times 0.67)]^{1/2}$

$= [0.073984 + 0.001024 + 0.0116634]^{1/2}$

$= 0.0866714^{1/2}$

= 0.2944, or approximately **29.4%.**

(Module 5.1, LOS 5.a)

16. (A) -12.0.

Explanation

The covariance is COV(XY) = ((0.3 × ((2 – 6) × (10 – 4))) + ((0.4 × ((6 – 6) × (2.5 – 4))) + (0.3 ×((10 – 6) × (0 – 4))) = –12.

(Module 5.1, LOS 5.b)

17. (C) Portfolio Z.

Explanation

Portfolio X: SFRatio = $\frac{12-5}{14}$ = 0.50

Portfolio Y: SFRatio = $\frac{17-5}{20}$ = 0.60

Portfolio Z: SFRatio = $\frac{22-5}{25}$ = 0.68

According to the safety-first criterion, Portfolio Z, with the largest ratio (0.68), is the best alternative.

(Module 5.1, LOS 5.c)

18. (A) 5.5%

Explanation

$[0.30 \times (0.15 - 0.066)^2 + 0.70 \times (0.03 - 0.066)^2]^{1/2} = 5.5\%$.

(Module 5.1, LOS 5.a)

19. (A) 10.04%.

Explanation

Find the weighted average return (0.10)(–5) + (0.30)(–2) + (0.50)(10) + (0.10)(31) = 7%.

Next, take differences, square them, multiply by the probability of the event and add the mup. That is the variance. Take the square root of the variance for Std. Dev. $(0.1)(-5-7)^2 + (0.3)(-2-7)^2 + (0.5)(10-7)^2 + (0.1)(31-7)^2$

= 100.8 = variance.

$100.8^{0.5} = 10.04\%$.

(Module 5.1, LOS 5.a)

20. (C) Portfolio 1.

Explanation

The probability of returns less than 5% can be minimized by selecting the portfolio with the greatest safety-first ratio using a threshold return of 5%:

Portfolio 1 = (9 – 5) / 5 = 4/5 = 0.80

Portfolio 2 = (8 – 5) / 4 = 3/4 = 0.75

Portfolio 3 = (7 – 5) / 3 = 2/3 = 0.67

(Module 5.1, LOS 5.c)

21. (C) 4.53%

Explanation

$E(R_A)$ = 11.65%

σ^2 = 0.0020506

$= 0.15(0.18 - 0.1165)^2 + 0.2(0.17 - 0.1165)^2 + 0.25 \quad (0.11 - 0.1165)^2 + 0.4(0.07 - 0.1165)^2$

$\sigma = 0.0452836$

(Module 5.1, LOS 5.a)

22. (B) Grant.

Explanation

Roy's safety-first ratios for the three portfolios:

Epps = (6 - 5) / 4 = 0.25

Flake = (7 - 5) / 9 = 0.222

Grant = (10 - 5) / 15 = 0.33

The portfolio with the largest safety-first ratio has the lowest probability of a return less than 5%. The investor should select the Grant portfolio.

(Module 5.1, LOS 5.c)

23. (C) 2.

Explanation

Roy's safety-first criterion requires the maximization of the SF Ratio:

SF Ratio = (expected return – threshold return) / standard deviation

Portfolio	Expected Return (%)	Standard Deviation (%)	SF Ratio
1	13	5	1.40
2	11	3	1.67
3	9	2	1.50

Portfolio #2 has the highest safety-first ratio at 1.67.

(Module 5.1, LOS 5.c)

24. (A) +0.65.

Explanation

The correlation coefficient

= Cov (X,Y) / [(Std Dev. X)(Std. Dev. Y)]

= 18.17 / 28 = 0.65

(Module 5.1, LOS 5.a)

25. (B) Expected returns.

Explanation

The correlations and standard deviations cannot give a measure of central tendency, such as the expected value.

(Module 5.1, LOS 5.a)

26. (B) 0.350.

Explanation

The correlation coefficient is:

Cov(A,B) / [(Std Dev A)(Std Dev B)]

$= 0.009 / [(\sqrt{0.02})(\sqrt{0.033})] = 0.350$.

(Module 5.1, LOS 5.a)

27. (A) 0.00144.

Explanation

$r = Cov(C,D) / (\sigma_C \times \sigma_D)$

$\sigma_C = (0.0009)^{0.5} = 0.03$

$\sigma_D = (0.0036)^{0.5} = 0.06$

$0.8(0.03)(0.06) = 0.00144$

(Module 5.1, LOS 5.a)

28. (C) 2.64%.

Explanation

The standard deviation of the portfolio is found by:

$[W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2\sigma_1\sigma_2\rho_{1,2}]^{0.5}$

$= [(0.40)^2(0.0015) + (0.60)^2(0.0021) + (2)(0.40)(0.60)(0.0387)(0.0458)(-0.35)]^{0.5}$

= 0.0264, or 2.64%.

(Module 5.1, LOS 5.a)

29. (A) 1.

Explanation

Roy's safety-first criterion requires the maximization of the SF Ratio:

SF Ratio = (expected return – threshold return) / standard deviation

Portfolio	Expected Return (%)	Standard Deviation (%)	SF Ratio
1	13	5	0.80
2	11	3	0.67
3	9	2	0.00

Portfolio #1 has the highest safety-first ratio at 0.80.

(Module 5.1, LOS 5.c)

30. (A) 6.20%.

Explanation

The standard deviation of the portfolio is found by:

$[W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2\sigma_1\sigma_2 r_{1,2}]^{0.5}$,

or $[(0.30)^2(0.046)^2 + (0.70)^2(0.078)^2 + (2)(0.30)(0.70)(0.046)(0.078)(0.45)]^{0.5}$
= 0.0620, or 6.20%.

(Module 5.1, LOS 5.a)

31. (A) 3.5%.

Explanation

In order to calculate the standard deviation of the company returns, first calculate the expected return, then the variance, and the standard deviation is the square root of the variance.

The expected value of the company return is the probability weighted average of the possible outcomes:

(0.20)(0.20) + (0.50)(0.15) + (0.30)(0.10) = 0.145.

The variance is the sum of the probability of each outcome multiplied by the squared deviation of each outcome from the expected return:

$(0.2)(0.20 - 0.145)^2 + (0.5)(0.15 - 0.145)^2 + (0.3)(0.1 - 0.145)^2$
= 0.000605 + 0.0000125 + 0.0006075 = 0.001225.

The standard deviation is the square root of 0.001225 = 0.035 or 3.5%.;

(Module 5.1, LOS 5.b)