Reading 57

THE TERM STRUCTURE OF INTEREST RATES - SPOT, PAR, & FORWARD CURVES

1. (A) equal to the market price of the bond.

Explanation

The value of a bond calculated using appropriate spot rates is its no-arbitrage value. If no arbitrage opportunities are present, this value is equal to the market price of a bond.

(Module 57.1, LOS 57.a)

2. (B) 5.73%.

Explanation

The spot rate is computed as follows:

$$\text{Spot rate}_{0,1} = \frac{(1+\text{Spot rate}_{0,2})^2}{(1+\text{forward rate}_{1,2})^1} - 1 = \frac{(1+0.05.89)^2}{(1+0.0605)^1} - 1 = 5.73\%$$

(Module 57.1, LOS 57.b)

3. (C) 100.10.

Explanation

This value is computed as follows:

Present Value = $6/1.05 + 6/1.055^2 + 106/1.06^3 = 100.10$

The value 95.07 results if the coupon payment at maturity of the bond is neglected.

(Module 57.1, LOS 57.a)

4. (C) zero-coupon bonds.

Explanation

A spot rate curve illustrates the yields for single payments to be made in various future periods, including short-term and long-term periods.

(Module 57.1, LOS 57.c)

5. (A) $76.

Explanation

The discount rate on an N-year, zero-coupon bond is the spot rate for Year N. Thus, find the spot rate in Year 3:

$(1 + Z_3)^3 = (1.0375) \times (1.095) \times (1.158) = 1.31556$

$Z_3 = (1.31556)^{1/3} - 1 = 0.0957 = 9.573\%$

Now, price this three-year, zero-coupon bond to yield 9.57%:

N = 3; I/Y = 9.57; FV = 100; CPT PV = –76.02 (ignore sign)

Hint: 100 / (1.0375 × 1.095 × 1.158) = 76.02 saves a couple of calculations.

(Module 57.1, LOS 57.b)

6. (A) $983.

Explanation

The value of the bond is simply the present value of discounted future cash flows, using the appropriate spot rate as the discount rate for each cash flow. The coupon payment of the bond is $40 (0.04 × 1,000). The bond price = $40/(1.02) + 1{,}040/(1.05)^2$ = $982.53.

(Module 57.1, LOS 57.a)

7. (B) $97.47.

Explanation

The bond price is computed as follows:

Bond price = $(5 / 1.0478) + (5 / 1.0556^2) + (105 / 1.0598^3)$ = $97.47

(Module 57.1, LOS 57.a)

8. (B) 6.57%.

Explanation

The four-year spot rate is computed as follows:

Four-year spot rate $= [(1 + 0.06)\ (1 + 0.065)\ (1 + 0.068)\ (1 + 0.07)]^{1/4} - 1 = 6.57\%$

(Module 57.1, LOS 57.b)

9. (A) 12.0%.

Explanation

$5_y1_y = [(1 + S_6)^6 / (1 + S_5)^5] - 1 = [(1.07)^6/(1.06)^5] - 1 = [1.5 / 1.338] - 1 = 0.12$

(Module 57.1, LOS 57.b)

10. (B) $9,694.

Explanation

We can calculate the price of the bond by discounting each of the annual payments by the appropriate spot rate and finding the sum of the present values. Price = $[1{,}500/(1.16)] + [11{,}500/(1.17)^2\ ]$ = \$9,694. Or, in keeping with the notion that each cash flow is a separate bond, sum the following transactions on your financial calculator:

N = 1, I/Y = 16.0, PMT = 0, FV = 1,500, CPT PV = 1,293

N = 2, I/Y = 17.0, PMT = 0, FV = 11,500, CPT PV = 8,401

Price = 1,293 + 8,401 = $9,694.

(Module 57.1, LOS 57.a)

11. (C) forward rate two years from today is 13.7%.

Explanation

The equation for the three-year spot rate, S_3, is $(1 + S_1)(1 + {}_{1y}1_y)(1 + {}_{2y}1_y) = (1 + S_3)^3$. Also, $(1 + S_1)(1 + {}_{1y}1_y) = (1 + S_2)^2$. So, $(1 + {}_{2y}1_y) = (1 + S_3)^3/ (1 + S_2)^2$, computed as: $(1 + 0.121)^3/(1 + 0.113)^2 = 1.137$. Thus, ${}_{2y}1_y = 0.137$, or 13.7%.

(Module 57.1, LOS 57.b)

12. (A) 6.25%.

Explanation

$S_4 = [\ (1.06)\ (1.062)\ (1.063)\ (1.065)\ ]^{25} - 1 = 6.25\%$.

(Module 57.1, LOS 57.b)

13. (C) time to maturity.

Explanation

The term structure of yield volatility refers to the relationship between yield volatility and time to maturity.

(Module 57.1, LOS 57.a)

14. (B) government spot rate that is specific to its maturity.

Explanation

To calculate a government bond's arbitrage-free value, each cash flow is discounted using the government spot rate that is specific to the maturity of the cash flow.

(Module 57.1, LOS 57.a)

15. (C) 8.02%.

Explanation

The forward rate is computed as follows:

One-year forward rate = $1.065^2 / 1.05 - 1 = 8.02\%$

(Module 57.1, LOS 57.b)

16. (C) 9.04%.

Explanation

$$\sqrt{\frac{(1.07)^4}{(1.05)^2}} - 1 = 0.0904, \text{or } \frac{(4x7)-(2x5)}{2} = 9 \text{ as an approximation}$$

(Module 57.1, LOS 57.b)

17. (B) 11.72%.

Explanation

$$4_y1_y = \frac{(1.055)^5}{(1.04)^4} - 1 = 0.1172$$

Note: 5(5.5) − 4(4) = 11.5%.

(Module 57.1, LOS 57.b)

18. (A) $103.17.

Explanation

The bond price is computed as follows:

Bond price = $6/1.055 + (102.50 + 6)/1.055^2 = \$103.17$

(Module 57.1, LOS 57.a)

19. (B) 5%.

Explanation

$6m6m/2 = [(1 + S_2/2)^2 / (1 + S_1/2)^1] - 1 = [(1.0225)^2/(1.02)^1] - 1$

$[1.0455 / 1.02] - 1 = 0.025$

$6m6m = 0.025 \times 2 = 0.05$

(Module 57.1, LOS 57.b)

20. (A) $912.

Explanation

We can calculate the price of the bond by discounting each of the annual payments by the appropriate spot rate and finding the sum of the present values. Price = [90 / (1.06)] + [90 / $(1.12)^2$] + [1,090 / $(1.13)^3$] = 912. Or, in keeping with the notion that each cash flow is a separate bond, sum the following transactions on your financial calculator:

N = 1; I/Y = 6.0; PMT = 0; FV = 90; CPT → PV = 84.91

N = 2; I/Y = 12.0; PMT = 0; FV = 90; CPT → PV = 71.75

N = 3; I/Y = 13.0; PMT = 0; FV = 1,090; CPT → PV = 755.42

Price = 84.91 + 71.75 + 755.42 = $912.08.

(Module 57.1, LOS 57.a)

21. (B) use of a series of spot interest rates that reflect the current term structure.

Explanation

The use of multiple discount rates (i.e., a series of spot rates that reflect the current term structure) will result in more accurate bond pricing and in so doing, will eliminate any meaningful arbitrage opportunities. That is why the use of a series of spot rates to discount bond cash flows is considered to be an arbitrage-free valuation procedure.

(Module 57.1, LOS 57.a)

22. (C) For a 3-year annual pay coupon bond, the first coupon can be discounted at 6.5%, the second coupon can be discounted at 7.0%, and the third coupon plus maturity value can be discounted at 9.2% to find the bond's arbitrage-free value.

Explanation

Spot interest rates can be used to price coupon bonds by taking each individual cash flow and discounting it at the appropriate spot rate for that year's payment. Note that the yield to maturity is the bond's internal rate of return that equates all cash flows to the bond's price. Current spot rates have nothing to do with the bond's yield to maturity.

(Module 57.1, LOS 57.a)

23. (C) $995.06.

Explanation

You need the find the present value of each cash flow using the spot rate that coincides with each cash flow.

The present value of cash flow 1 is: FV = $55; PMT = 0; I/Y = 5.2%; N = 1; CPT → PV = -$52.28.

The present value of cash flow 2 is: FV = $55; PMT = 0; I/Y = 5.5%; N = 2; CPT → PV = – $49.42.

The present value of cash flow 3 is: FV = $1,055; PMT = 0; I/Y = 5.7%; N = 3; CPT → PV = – $893.36.

The most you pay for the bond is the sum of: $52.28 + $49.42 + $893.36 = $995.06.

(Module 57.1, LOS 57.a)

24. (C) 10%.

Explanation

Implied 1-year forward rate in four years =

$$\frac{(1+S_5)^5}{(1+S_4)^4}-1=\frac{1.08^5}{1.075^4}-1=\frac{1.4693}{1.3355}-1=0.1002 \text{ or } 10.02\%.$$

Alternatively, 5 × 8%

(Module 57.1, LOS 57.b)

25. (B) 8.61%.

Explanation

The forward rate is computed as follows:

$$\text{Forward rate}_{1,2}=\frac{(1+\text{Spot rate}_{0,2})^2}{(1+\text{Spot rate}_{0,1})^1}-1=\frac{(1+0.0732)^2}{(1+0.0605)^1}-1=8.61\%$$

(Module 57.1, LOS 57.b)

26. (C) issuers.

Explanation

Yield curves are typically constructed for bonds of the same or similar issuers, such as a government bond yield curve or AA rated corporate bond yield curve.

(Module 57.1, LOS 57.c)

27. (A) 4.5%.

Explanation

$(1.04^5 / 1.032^2)^{1/3} - 1 = 4.5\%$.

(Module 57.1, LOS 57.b)

28. (C) 7.0%.

Explanation

$(1 + {}_{1y}1_{y})(1 + s_1) = (1 + s_2)^2$

$(1 + 0.05)(1 + s_1) = (1 + 0.06)^2$

$(1 + s_1) = (1.06)^2 / (1 + 0.05)$

$1 + s_1 = 1.1236 / 1.05$

$1 + s_1 = 1.0701$

$s_1 = 0.07$ or 7%

(Module 57.1, LOS 57.b)

29. (A) $966.

Explanation

We can calculate the price of the bond by discounting each of the annual payments by the appropriate spot rate and finding the sum of the present values. Bond price = $[60 / (1.05)] + [1{,}060 / (1.08)^2] = \$966$. Or, in keeping with the notion that each cash flow is a separate bond, sum the following transactions on your financial calculator:

N = 1; I/Y = 5.0; PMT = 0; FV = 60; CPT → PV = 57.14

N = 2; I/Y = 8.0; PMT = 0; FV = 1,060; CPT → PV = 908.78

Price = 57.14 + 908.78 = $966.

(Module 57.1, LOS 57.a)

30. (C) 12.0%.

Explanation

$[(1 + S_4)^4 / (1 + S_3)^3] - 1 = 12.01\% = 12\%$.

(Module 57.1, LOS 57.b)

31. (C) Zero-coupon bond yield curve.

Explanation

The spot rate yield curve shows the appropriate rates for discounting single cash flows occurring at different times in the future. Conceptually, these rates are equivalent to yields on zero-coupon bonds. The par bond yield curve shows the YTMs at which bonds of various maturities would trade at par value. Forward rates are expected future short-term rates.

(Module 57.1, LOS 57.c)

32. (B) 6.92%.

Explanation

First determine the current price of the bond:

$= 6 / 1.05 + 6 / (1.06)^2 + 106 / (1.07)^3 = 5.71 + 5.34 + 86.53 = 97.58$

Then compute the yield of the bond:

N = 3; PMT = 6; FV = 100; PV = -97.58; CPT → I/Y = 6.92%

(Module 57.1, LOS 57.a)

33. (B) yield-to-maturity on a 10-year coupon bond.

Explanation

A 10-year spot rate is the yield-to-maturity on a 10-year zero-coupon security, and is the appropriate discount rate for the year 10 cash flow for a 20-year (or any maturity greater than or equal to 10 years) bond. Spot rates are used to value bonds and to ensure that bond prices eliminate any possibility for arbitrage resulting from buying a coupon security, stripping it of its coupons and principal payment, and reselling the strips as separate zero coupon securities. The yield to maturity on a 10-year bond is the (complex) average of the spot rates for all its cash flows.

(Module 57.1, LOS 57.a)

34. (B) $700.

Explanation

Based on the given spot and forward rates, the 4-year spot rate equals $[(1.07)(1.0815)(1.103)(1.120)]^{1/4} - 1 = 9.35\%$.

Bond value: N = 4; FV = 1,000; I/Y = 9.35; PMT = 0; CPT → PV = –699.40

(Module 57.1, LOS 57.a)

35. (A) 6.81%.

Explanation

$(1 + S_3)^3 = (1 + S_2)^2(1 + 2y1y)$

$(1 + 2y1y) = (1 + S_3)^3 / (1 + S_2)^2$

$(1 + 2y1y) = (1.0611)^3 / (1.0576)^2 = 1.0681$

$2y1y = 6.81\%$

(Module 57.1, LOS 57.b)

36. (C) $19.22.

Explanation

The no-arbitrage price of a bond is determined by discounting each of its cash flows at the appropriate spot rate. Any difference between the no-arbitrage price and the market price of a bond represents a potential arbitrage profit.

$$= \frac{20}{1.01} + \frac{20}{1.0125^2} + \frac{20}{1.015^3} + \frac{20}{1.02^4} + \frac{1020}{1.03^5}$$

$$= 19.80 + 19.51 + 19.13 + 18.48 + 879.86 = \$956.78$$

$$976 - 956.78 = \$19.22$$

(Module 57.1, LOS 57.a)

37. (B) Price appreciation creates all of the zero-coupon bond's return.

Explanation

Zero-coupon bonds are quite special. Because zero-coupon bonds have no coupons (all of the bond's return comes from price appreciation), investors have no uncertainty about the rate at which coupons will be invested. Spot rates are defined as interest rates used to discount a single cash flow to be received in the future. If the yield to maturity on a 2-year zero is 6%, we can say that the 2-year spot rate is 6%.

(Module 57.1, LOS 57.a)