## Reading 59

## YIELD-BASED BOND DURATION MEASURES \& PROPERTIES

1. (A) 24-year maturity, $5.0 \%$ coupon.

Explanation
Interest rate risk (or price volatility) increases at longer maturities and with lower coupons.
(Module 59.1, LOS 59.b)
2. (B) low coupon and a long maturity.

## Explanation

Other things equal, a bond with a low coupon and long maturity will have the greatest price volatility.
(Module 59.1, LOS 59.b)
3. (C) €25 million.

Explanation
Money duration is expressed in currency units.
(Module 59.1, LOS 59.a)
4. (C) $\$ 0.82$.

Explanation
PVBP = initial price - price if yield changed by 1 bps.
Initial price: Price with change:

| FV $=1000$ | FV $=1000$ |
| :---: | :---: |
| PMT $=80$ | PMT $=80$ |
| $\mathrm{~N}=18$ | $\mathrm{~N}=18$ |
| $\mathrm{I} / \mathrm{Y}=9 \%$ | $\mathrm{I} / \mathrm{Y}=9.01$ |

CPT PV $=912.44375$ CPT PV $=911.6271$
PVBP $=912.44375-911.6271=0.82$
(Module 59.1, LOS 59.a)
5. (B) Zero-coupon bond.

Explanation
The Macaulay duration of a zero-coupon bond is equal to its time to maturity. Its price is greatly affected by changes in interest rates because its only cash-flow is at maturity and is discounted from the time at maturity until the present.
(Module 59.1, LOS 59.b)
6. (A) decreases.

## Explanation

The higher the yield on a bond the lower the price volatility (duration) will be. When interest rates increase the price of the bond will decrease and the yield will increase because the current yield = (annual cash coupon payment) / (bond price). As the bond price decreases the yield increases and the price volatility (duration) will decrease.
(Module 59.1, LOS 59.b)
7. (B) 12.8\%.

## Explanation

Approximate modified duration $=$ (price if yield down - price if yield up) / ( $2 \times$ initial price $\times$ yield change expressed as a decimal).
Here, the initial price is par, or $\$ 1,000$ because we are told the bond was issued today at par. So, the calculation is: $(1039.59-962.77) /(2 \times 1000 \times 0.003)$ = 76.82 / $6.00=12.80$.
(Module 59.1, LOS 59.a)
8. (C) underestimates the increase in price for decreases in yield.

Explanation
For large changes in yield, duration underestimates the increase in price when yield decreases and overestimates the decrease in price when yield increases. This is because duration is a linear estimate that does not account for the convexity (curvature) in the price/yield relationship.
(Module 59.1, LOS 59.a)
9. (C) yield to maturity is lower.

## Explanation

In this case the only determinant that will cause higher interest rate risk is having a low yield to maturity. A higher coupon rate and a higher current yield will result in lower interest rate risk.
(Module 59.1, LOS 59.b)
10. (A) $\$ 9.6$ million.

Explanation
Money duration $=$ annual modified duration $\times$ portfolio value $=8 \times \$ 12$ million = \$96,000,000.
(Module 59.1, LOS 59.a)
11. (B) 15-year, $8 \%$ coupon bond.

## Explanation

If bonds are identical except for maturity and coupon, the one with the longest maturity and lowest coupon will have the greatest duration. The later the cash flows are received, the greater the duration.
(Module 59.1, LOS 59.b)
12. (B) ten year, option-free 4\% coupon bond.

## Explanation

If two bonds are identical in all respects except their term to maturity, the longer term bond will be more sensitive to changes in interest rates. All else the same, if a bond has a lower coupon rate when compared with another, it will have greater interest rate risk. Therefore, for the option-free bonds, the 10 year 4\% coupon is the longest term and has the lowest coupon rate. The call feature does not make a bond more sensitive to changes in interest rates, because it places a ceiling on the maximum price investors will be willing to pay. If interest rates decrease enough the bonds will be called.
(Module 59.1, LOS 59.b)
13. (A) 10.03.

## Explanation

Calculate the new bond prices at the 50 basis point change in rates both up or down and then plug into the approximate modified duration equation:
Current price: $\mathrm{N}=50 ; \mathrm{FV}=1,000 ; \mathrm{PMT}=(0.075 / 2) \times 1,000=37.50$;
$\mathrm{I} / \mathrm{Y}=4.625 ; \mathrm{CPT} \rightarrow \mathrm{PV}=\$ 830.54$.
+50 basis pts: $\mathrm{N}=50 ; \mathrm{FV}=1,000 ; \mathrm{PMT}=(0.075 / 2) 1,000=37.50$;
$\mathrm{I} / \mathrm{Y}=4.875 ; \mathrm{CPT} \rightarrow \mathrm{PV}=\$ 790.59$.
-50 basis pts: $\mathrm{N}=50 ; \mathrm{FV}=1,000 ; \mathrm{PMT}=(0.075 / 2) 1,000=37.50$;
$\mathrm{I} / \mathrm{Y}=4.375 ; \mathrm{CPT} \rightarrow \mathrm{PV}=\$ 873.93$.
Approximate modified duration $=(873.93-790.59) /(2 \times 830.54 \times 0.005)$
$=10.03$.

## (Module 59.1, LOS 59.a)

14. (C) -\$61.10.

## Explanation

First, compute the current price of the bond as: $\mathrm{FV}=1,000 ; \mathrm{PMT}=55 ; \mathrm{N}=10$; $\mathrm{I} / \mathrm{Y}=4.7 ; \mathrm{CPT} \rightarrow \mathrm{PV}=-1,062.68$. Then compute the price of the bond if rates rise by 75 basis points to $5.45 \%$ as: $F V=1,000 ; P M T=55 ; N=10 ; \mathrm{I} / \mathrm{Y}=5.45$; $\mathrm{CPT} \rightarrow \mathrm{PV}=-1,003.78$. Then compute the price of the bond if rates fall by 75 basis points to $3.95 \%$ as: $\mathrm{FV}=1,000 ; \mathrm{PMT}=55 ; \mathrm{N}=10 ; \mathrm{I} / \mathrm{Y}=3.95$; CPT $\rightarrow$ $\mathrm{PV}=-1,126.03$.
The formula for approximate modified duration is: $\left(\mathrm{V}_{-}-\mathrm{V}_{+}\right) /\left(2 \mathrm{~V}_{0} \Delta \mathrm{y}\right)$. Therefore, modifiedduration is: $(\$ 1,126.03-\$ 1,003.78) /(2 \times \$ 1,062.68 \times 0.0075)=7.67$. The formula for the percentage price change is then: -(duration)( $\Delta \mathrm{YTM}$ ). Therefore, theestimated percentage price change using duration is: $(7.67)(0.75 \%)=-5.75 \%$. The estimated price change is then: $(-0.0575)$ (\$1,062.68) = -\$61.10
(Module 59.1, LOS 59.a)
15. (B) $\$ 0.64$.

Explanation
PVBP = initial price - price if yield changed by 1 bps.
Initial price: Price with change:

| $\mathrm{FV}=1000$ | $\mathrm{FV}=1000$ |
| :---: | :---: |
| $\mathrm{PMT}=80$ | $\mathrm{PMT}=80$ |
| $\mathrm{~N}=18$ | $\mathrm{~N}=18$ |
| $\mathrm{I} / \mathrm{Y}=9 \%$ | $\mathrm{I} / \mathrm{Y}=9.01$ |

CPT PV $=1225.92$ CPT $\mathrm{PV}=1225.28$
$\mathrm{PVBP}=1,225.92-1,225.28=0.64$
(Module 59.1, LOS 59.a)
16. (B) only one of these effects.

## Explanation

The analyst is incorrect with respect to coupon rate. As the coupon rate decreases, the interest rate risk of a bond increases. Lower coupons cause greater relative weight to be placed on the principal repayment. Because this cash flow occurs farther out in time, its present value is much more sensitive to changes in interest rates. As the coupon rate goes to zero (i.e., a zero-coupon bond), all of the bond's return relies on the return of principal which as stated before is highly sensitive to interest rate changes. The analyst is correct with respect to maturity. As the maturity of a bond increases, an investor must wait longer for the eventual repayment of the bond principal. As the length of time until principal payment increases, the probability that interest rates will change increases. If interest rates increase, the present value of the final payment (which is the largest cash flow of the bond) decreases. At longer maturities, the present value decreases by greater amounts. Thus, interest rate risk typically increases as the maturity of the bond increases. (The exception is for long-term discount bonds, which may exhibit a range of long maturities over which an increase in maturity decreases interest rate risk.)
(Module 59.1, LOS 59.b)
17. (C) $\$ 0.50$.

## Explanation

First we compute the yield to maturity of the bond. $\mathrm{PV}=-\$ 958.97, \mathrm{FV}=\$ 1,000$, PMT $=\$ 21, \mathrm{~N}=12$, CPT $\mathrm{I} / \mathrm{Y}=2.5 \%$, multiply by 2 since it is a semiannual bond to get an annualized yield to maturity of $5.0 \%$. Now compute the price of the bond at using yield one basis point higher, or $5.01 \%$. $\mathrm{FV}=\$ 1,000, \mathrm{PMT}=21, \mathrm{~N}$ $=12, \mathrm{I} / \mathrm{Y}=(5.01 / 2=) 2.505$, CPT $\mathrm{PV}=-\$ 958.47$. The price changes from $\$ 958.97$ to \$958.47, or \$0.50.
(Module 59.1, LOS 59.a)
18. (B) 10.6.

## Explanation

If the yield on the bond were $7.25 \%$, the price would be 97.402 and would be 102.701 if the yield were $6.75 \%$. The approximate modified duration for this bond based on a 25 basis point change in yield is calculated as:
$\frac{102.701-97.402}{2(100)(0.0025)}=10.5976$
(Module 59.1, LOS 59.a)
19. (A) 5.37.

## Explanation

Approximate modified duration is computed as follows:
Duartion $=\frac{105.50-100}{2 \times 102.50 \times 0.005}=5.37$

## (Module 59.1, LOS 59.a)

20. (B) 4.33.

## Explanation

Modified duration is a measure of a bond's sensitivity to changes in interest rates.
Approximate modified duration $=\left(\mathrm{V}_{-}-\mathrm{V}_{+}\right) /\left[2 \mathrm{~V}_{0}\right.$ (change in required yield) $]$ where:
V - = estimated price if yield decreases by a given amount
$\mathrm{V}_{+}=$estimated price if yield increases by a given amount
$\mathrm{V}_{0}=$ initial observed bond price
Thus, duration $=(104.45-95.79) /(2 \times 100 \times 0.01)=4.33$. Remember that the change in interest rates must be in decimal form.
(Module 59.1, LOS 59.a)
21. (A) $\$ 606$.

Explanation
Money duration per $\$ 100$ par value $=$ annual modified duration $\times$ full price per $\$ 100$ par value $=6.1 \times \$ 99.30=\$ 605.73$
(Module 59.1, LOS 59.a)
22. (B) decreases increases

Explanation
As coupon rates increase the duration on the bond will decrease because investors are receiving more cash flow sooner. As maturity increases, duration will increase because the payments are spread out over a longer period of time.
(Module 59.1, LOS 59.b)
23. (B) Inclusion of a call feature.

## Explanation

Inclusion of a call feature will decrease the duration of a fixed income security. The other choices increase duration.
(Module 59.1, LOS 59.b)
24. (A) less price volatility at higher yields.

## Explanation

The only true statement is that putable bonds will have less price volatility at higher yields. At higher yields the put becomes more valuable and reduces the decline in price of the putable bond relative to the option-free bond. On the other hand, when yields are low, the put option has little or no value and the putable bond will behave much like an option-free bond. Therefore at low yields a putable bond will not have more price volatility nor will it have less price volatility than a similar option-free bond.
(Module 59.1, LOS 59.b)
25. (B) Par value government bond maturing in five years.

Explanation
The bond with the least percentage price change will be the bond with the lowest interest rate risk. Higher coupons or shorter maturities decrease interest rate risk. The coupon paying bond with only five years to maturity will have the lowest interest rate risk.
(Module 59.1, LOS 59.b)
26. (B) increases the bond's duration, increasing price risk.

Explanation
A call provision decreases the bond's duration because a call provision introduces prepayment risk that should be factored in the calculation. For the investor, one of the most significant risks of callable (or prepayable) bonds is that they can be called/retired prematurely. Because bonds are nearly always called for prepayment after interest rates have decreased significantly, the investor will find it nearly impossible to find comparable investment vehicles. Thus, investors have to replace their high-yielding bonds with much loweryielding issues. From the bondholder's perspective, a called bond means not only a disruption in cash flow but also a sharply reduced rate of return.
Generally speaking, the following conditions apply to callable bonds:

- The cash flows associated with callable bonds become unpredictable, since the life of the bond could be much shorter than its term to maturity, due to the call provision.
- The bondholder is exposed to the risk of investing the proceeds of the bond at lower interest rates after the bond is called. This is known as reinvestment risk.
- The potential for price appreciation is reduced, because the possibility of a call limits or caps the price of the bond near the call price if interest rates fall.
(Module 59.1, LOS 59.b)

27. (C) zero-coupon bond.

Explanation
The duration of a zero-coupon bond is equal to its time to maturity since the only cash flows made is the principal payment at maturity of the bond. Therefore, it has the highest interest rate sensitivity among the four securities.
A floating rate bond is incorrect because the duration, which is the interest rate sensitivity, is equal to the time until the next coupon is paid. So this bond has a very low interest rate sensitivity.
A coupon bond with a coupon rate of $5 \%$ is incorrect because the duration of a coupon paying bond is lower than a zero-coupon bond since cash flows are made before maturity of the bond. Therefore, its interest rate sensitivity is lower.
(Module 59.1, LOS 59.b)
28. (A) is lower.

## Explanation

Modified duration = Macaulay duration / (1 + YTM). Modified duration is lower than Macaulay duration unless YTM equals zero.
(Module 59.1, LOS 59.a)
29. (B) 10-year maturity, 10\% coupon rate.

## Explanation

All else constant, a bond with a longer maturity will be more sensitive to changes in interest rates. All else constant, a bond with a lower coupon will have greater interest rate risk.
(Module 59.1, LOS 59.b)
30. (B) 7.66.

Explanation
The change in the yield is 30 basis points.
Approximate modified duration $=(98.47-94.06) /(2 \times 96.00 \times 0.003)=7.6563$.
(Module 59.1, LOS 59.a)
31. (A) 4.187.

## Explanation

The YTM on the bond is $6.5 \% . \mathrm{N}=5, \mathrm{PV}=-979.22, \mathrm{PMT}=60, \mathrm{FV}=1,000, \mathrm{CPT}$ $\mathrm{I} / \mathrm{Y}=6.5 \%$
Modified duration $=$ Macaulay duration $/(1+Y T M)=4.4587 / 1.065=4.187$.
(Module 59.1, LOS 59.a)
32. (C) Bonds with higher coupons have lower interest rate risk.

Explanation
Other things equal, bonds with higher coupons have lower interest rate risk. Note that the other statements are false. Bonds with longer maturities have higher interest rate risk. Callable bonds have a ceiling value as yields decline.
(Module 59.1, LOS 59.b)


